This article has been accepted for publication in IEEE Transactions on Aerospace and Electronic Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TAES.2022.3193948

Deep ToA Mask Based Recursive Radar Pulse Deinterleaving

Haoran Xiang, Furao Shen, *Member, IEEE,* and Jian Zhao, *Senior Member, IEEE*

***Abstract*—In a complex electromagnetic environment, multiple radar signals of various modes are densely interleaved. In this environment, radar parameters overlap seriously and change continuously over time. Traditional radar pulse deinterleaving algorithms face severe challenges such as parameters missing, pulse jitter, and the increasing number of electronic countermeasure devices. In this paper, we propose a recursive deinterleaving algorithm based on blind signal separation and deep learning to cope with such a situation. The Recursive Deinterleaving Network (RDN) of Deep ToA Mask (DTM) maps the ToA train to a suitable feature space first. ToA coefficient masks of each radar emitter are estimated with the local and global context information of the radar pulse feature. Then the RDN sorts out several radar pulse trains recursively with the help of dual-path attention. It also predicts the number of emitters with nearly 100% accuracy and handles the unknown Pulse Repetition Interval (PRI) situation. More accurate pulse deinterleaving results can be obtained if the DTM utilizes more radar parameters through proper preprocessing fine-tuning and post-processing re-clustering. The processing steps of the DTM are introduced in detail. The simulation shows it can achieve 97% sorting accuracy for multipulse interleaved radar train with jitter PRI and pulse missing. The DTM algorithm can also deal with the interleaved radar signals of different PRI modulations by re-clustering with noisy PDW information. On the premise of knowing the modulation type or PRI information, the pulse train deinterleaving accuracy of multi-modulation emitters is higher.**

***Index Terms*—Blind Radar Pulse Sorting, Deep ToA Mask, Recursive Deinterleaving Network**

I. INTRODUCTION

ELECTRONIC warfare (EW) has become a vital part of modern warfare, and Electronic Support Measure (ESM) plays an important role in EW [1]. The function of the ESM system is to intercept and analyze electromagnetic signals, and quickly identify threatening signal sources [2]. The deinterleaving of radar signals is a key technology in the processing of ESM systems. After deinterleaving, the signals belonging to different sources are sorted from the mixed radar pulse train, and then different tasks such as signal source classification, pattern recognition, and tracking are performed [3].

The pulse train received by the ESM system is described by Pulse Description Words (PDW). PDW usually includes five basic parameters: Pulse Width (PW), Radio Frequency (RF), Pulse Amplitude (PA), Direction of Arrival (DoA), and Time of Arrival (ToA) [4]. Among them, the first-order difference of the ToA train is also called Pulse Repetition Interval (PRI) if the PDWs are already deinterleaved [5], which is the intrinsic property of the pulse train. Among radar signal characteristic parameters, PRI is one of the parameters with The most diverse way of working, the largest parameter range, and the fastest parameter change. The recognition of pulse train modulation patterns becomes more challenging with factors such as the high complexity of radar operating modes, high-density electromagnetic signal spaces, extremely complex signal parameters, and low probability of signal interception. In such a complex environment, various parameters of the PDW have a certain probability of being lost or interfered in the measurement process, which makes the difficulty of radar signal deinterleaving rapidly increase [6].

PRI modulation of radar pulse signal is an important part to analyze and identify radar functions and applications. Even for the same radar, there may be several or even a dozen modulation types. With the introduction of new electronic anti-reconnaissance and radar technologies, the modulation type of radar radiation sources has become more and more complex. PRI modulation and coding makes it more difficult to cheat radar jamming in radar countermeasure because the enemy will not be able to know or predict the accurate structure of the transmission waveform. The PRI modulation of intra pulse coding can compress complex signals and effectively improve the ability of target detection. It has high average power and low peak power. Its wide bandwidth can improve the range resolution and reduce the reflection of chaff passive interference. Due to its low peak power, the radiated signal is not easy to be detected by the enemy's electronic support measures. Therefore, the pulse compression radar using this kind of complex signal has better electronic anti countermeasure performance. However, if the PRI modulation type can be correctly identified, it will be easier to identify the threat radiation source [7]. PRI modulation identification can usually be achieved only by the ToA feature [8]. This paper mainly focuses on the radar sorting by the ToA.

To ensure accuracy in practical applications, the current signal sorting method uses one or more of the five PDW parameters for pulse deinterleaving. Traditional radar pulse sorting algorithms, such as Cumulative Difference Histogram Method (CDIF) [9], Sequence Difference Histogram Method (SDIF) [10] and Improved Histogram Method (IHM) [11], have clear and intuitive theories, but are not robust to environmental noise and PRI jitter and missing. PDW-based clustering methods include K-means [12], fuzzy clustering [13], support

---

Author's address: Haoran Xiang and Furao Shen are with National Key Laboratory for Novel Software Technology, School of Artificial Intelligence, Nanjing University, Nanjing 210023, China (e-mail: mg20370040@smail.nju.edu.cn; frshen@nju.edu.cn). Jian Zhao is with School of Electronic Science and Engineering, Nanjing University, Nanjing 210023, China (e-mail: jianzhao@nju.edu.cn). *(Corresponding Authors: Furao Shen and Jian Zhao)*

vector clustering [14], hierarchical clustering[15] and Gaussian distribution based clustering[16, 17]. This type of method is mainly based on the assumption that the same radar has similar parameter distribution, and clusters are performed by designing the distance between PDW features. However, the clustering method mainly focuses on the similarity of parameters, so it may not complete the target task well. Gasperini and Stefano [18] have proposed to encode radar signals into images, and then perform clustering based on image segmentation. However, there are also problems such as low time resolution of clustering and difficulty in determining the number of clustered radars. The prediction method based on the feature trajectory [19] mainly uses the RNN [20, 21] that is combined with the PA to perform signal sorting. It is inspired by the Kalman filter and the Hough transform sorting method [22]. It can achieve good results when the PDW parameters used are accurately measured. When any of these parameters are missing or misleading interference occurs, the result will fluctuate. The method based on the Denoise Autoencoder (DA) [23] can sort the ToA train of multiple PRI modes well. However, those algorithms perform well only when the radar PRI value and the number of radar emitters are known. In an actual environment, that knowledge is not available in most cases. The PRI transformation method [24] is a traditional single-parameter deinterleaving method. It seems that enough radar information is not provided because only ToA information can be used. The ToA train is a sparse signal, and the mixed signal is easy to be sorted in suitable feature space (such as the PRI autocorrelation transform domain) [25]. Even if multiple radar pulse trains are mixed, they can be deinterleaved through their spectral information. The traditional PRI transformation method first needs other PDW parameters excluding ToA for pre-sorting [26]. In the sorting process, not only a high time resolution (about 10ns), and calculation of the autocorrelation function, but also Sequence Search (SS) and artificial thresholds are required. Sequence Search refers to using the estimated PRI information to sequentially search the radar pulse points consistent with the current estimated PRI, and estimate the PRI more accurately according to the search results. This method processes the ToA train by guessing and verifying the PRI at a lower speed. Using the same threshold in different environments will result in an accuracy decrease. Deep neural network based algorithm [27, 28] is better for modeling the complex blind radar pulse sorting problem with only the ToA feature.

The Deep ToA Mask (DTM) algorithm proposed in this paper is mainly based on the blind source signal separation theory [29, 30], inspired by methods such as Independent Component Analysis (ICA) [31] and Non-negative Matrix Factorization (NMF) [32]. Recursive Deinterleaving Network (RDN) is the core of it. Blind source signal separation algorithms can separate each source by relying on a priori information or statistical methods when only the mixed signal sequence is given. Because different source signals have their own characteristics. In the radar pulse signal, there is also a specific pattern in the pulse sequence emitted by each radar emitter, especially the periodicity brought by PRI. On the other hand, the pulse sequence is generated by a series of pulse parameters. If the parameters of each source sequence can be implicitly estimated by neural network combined with supervised learning, then radar pulse deinterleaving by ToA alone would be feasible. Since multiple source sequences with different characteristics are mixed together, their characteristics are difficult to observe in the time domain. Therefore, we want to map the radar pulse sequence to the latent space defined by the signal itself, which is similar to the time-frequency domain, and separate it in latent space. Similar ideas are often used in the field of speech separation and have been shown to be feasible [33].

Looking back at previous work, it can be seen that most algorithms use multiple features in the PDW. However, in practical applications, we find that not every feature in the PDW is always available. Even when it is available, it may not be used for sorting due to noise. Directly using the noisy or missing PDW features for sorting is very ineffective. Since every radar pulse train has the ToA feature, we want to use only ToA feature to deinterleave radar pulse trains in complex battlefield situations and get high-precision results. When other features are available, we can also incorporate them into our algorithm. We will focus on the sorting accuracy of the DTM algorithm in the scenario of multi-emitter radar pulse sequences where four PRI modulation types, jitter PRI, stagger PRI, sliding PRI and group PRI appear randomly. We also discuss how to incorporate certain information, such as modulation type or noisy PDW data, to apply this deinterleaving method to complex multi-modulation multi-emitter scenario.

The main contributions of this paper are summarized as follows:

- To the best of our knowledge, this is the first deep learning method combining the deep separable dilated convolution [33] and dual-path attention mechanism to constructs the ToA mask of each emitter in the encoder-decoder feature space. The recursive deinterleaving method can improve the sorting accuracy and better solve the channel permutation problem. The accuracy of the DTM is much higher than methods based on sequence search, clustering and other deep learning deinterleaving. Moreover, the DTM algorithm only requires a short radar pulse sequence of about ten minutes during training, and it can achieve real-time during inference,
- We only make use of the least and most essential feature, that is, the ToA train, to get a robust radar pulse deinterleaving result and predict the emitter number accurately. It has good generalization. The DTM algorithm does not require prior knowledge of PRI and the number of radar emitters, which is more conducive to actual sorting. When more credible PDW parameters are available, we also studied how to incorporate them into the DTM algorithm framework. We also propose pre-processing fine-tuning and post-processing re-clustering methods to make full use of that extra information. It can automatically adapt to the complex multi-emitter environment, and there is no need to manually set the threshold in inference.
- Without any prior knowledge of the PRI modulation mode, the DTM algorithm can still deinterleave modulated radar sequence emitted by uncertain number of radar

emitters with an accuracy of 90.83%. Moreover, we also consider three possible ways to further improve the effect of the DTM algorithm in the multiple PRI modulations scenario. Simulation proves that in the case of known PRI or unknown PRI with known modulation mode types, the DTM algorithm can further sort radar signals with a high accurate under complex modulation conditions such as stagger PRI, sliding PRI, group PRI and so on.

II. Problem Formalization

A discrete radar PDW train $\boldsymbol{D}$ can be formally expressed as $\boldsymbol{D} = [\boldsymbol{d_1}, \boldsymbol{d_2}, ..., \boldsymbol{d_T}]$. Among them, d_i is a multi-dimensional vector and contains some PDW parameters. The traditional method searches the pulse sequence according to possible PRIs [9–11]. This method requires the manual setting of the search threshold, but there is no suitable threshold for all the EW environments. In addition, the PRI is not fixed in the actual environment. Pulse missing and jitter often occur in this condition. At this time, the accuracy of the method based on Sequence Search or deep learning prediction will drop sharply. For the adaptive neural network-based clustering methods [34], they regard each $\boldsymbol{d_i}$ of the PDW train $\boldsymbol{D}$ as an independent random variable with multiple dimensions, and ignore the sequential nature of $\boldsymbol{D}$. Although the method based on Long Short-Term Memory (LSTM) deinterleaving [20, 35] considers the timing relationship, it is more sensitive to the starting point of the sequence. It is not strong in generalization and is slow. Therefore, these methods are difficult to apply to a real-time electromagnetic environment.

A. Representation of ToA Pulse Train

This paper mainly studies how to do deinterleaving of radar pulse trains only through the ToA train itself. Take the ToA train $\boldsymbol{S}$ from the PDW train $\boldsymbol{D}$. Now $\boldsymbol{S}$ is a dense vector, and each element S_i represents a ToA point. Also, $\boldsymbol{S}$ is a single channel signal mixed from multiple radar ToA trains. Relying on single channel signal to predict the value of multiple channels sequences is an underdetermined problem. Fortunately, by transforming a raw ToA train to a sparse radar pulse sequence, pulse deinterleaving can be achieved using the sparsity of $\boldsymbol{S}$. Regard $\boldsymbol{S}$ as an ordered set in ascending order of ToA, i.e., $\boldsymbol{S} = \{ToA_1, ToA_2, ..., ToA_n\}$. For each $ToA_i \in \boldsymbol{S}$, define $ToA_i \triangleq ToA_i - ToA_1 + 1$. Here, $\triangleq$ is a assignment symbol. And we want to scale the value of $\boldsymbol{S}$ into $[1, T]$. Therefore, after scaling, $ToA_1 = 1$, and $ToA_n = T$. The ToA sequence is determined mainly by the PRI. If the ToA sequence is known, the PRI can be estimated by:

$$PRI(i) = ToA_{i+1} - ToA_i. \tag{1}$$

We can convert the set $\boldsymbol{S}$ into a pulse sequence $\boldsymbol{x} = [x_1, x_2, \ldots, x_n, \ldots, x_T]$ in the time domain as follows:

$$\boldsymbol{x}(n) = \sum_{toa \in \boldsymbol{S}} \delta(n - toa). \tag{2}$$

where $\delta(n - toa)$ represents the shifted unit impulse function at toa, and n is the time index of the pulse sequence $\boldsymbol{x}$. After the above conversion, $\boldsymbol{x}$ is a interleaved pulse sequence. Assuming that $\boldsymbol{x}$ is obtained by mixing K radar pulse trains, we define y as radar emitter numbers $[1, K]$ and a mapping $f : \boldsymbol{x} \to y$, where K is generally unknown in advance. Define $\boldsymbol{x_k}$ as a pulse sequence emitted by the k-th radar eimitter, and define $\hat{\boldsymbol{x_k}}$ as the pulse sequence deinterleaved by any sorting algorithm. Similarly:

$$\boldsymbol{x_k}(n) = \sum_{toa_k \in \boldsymbol{S} \wedge f(\boldsymbol{x}(n)) = k} \delta(n - toa_k). \tag{3}$$

When the deinterleaving mapping $f : \boldsymbol{x} \to y$ is known, $\hat{\boldsymbol{x}}_{\boldsymbol{k}}$ can be separated from $\boldsymbol{x}$ easily. Therefore, the problem is how to solve the mapping f from $\boldsymbol{x}$ using sorting algorithm. $\hat{\boldsymbol{x}}$ can be reconstructed by $\hat{\boldsymbol{x}}_{\boldsymbol{k}}$, that is, $\hat{\boldsymbol{x}} = \sum_{k=1}^{K} \hat{\boldsymbol{x}}_{\boldsymbol{k}}$. If the interleaved radar sequence is not influenced by noise and the mapping f is accurate, then $\hat{\boldsymbol{x}} = \boldsymbol{x}$. But if $\boldsymbol{x} = \sum_{k=1}^{K} \boldsymbol{x_k} + Noise$, we have $E(\boldsymbol{x}) = E(\hat{\boldsymbol{x}})$, where $Noise$ represents the Gaussian noise, and $E(\boldsymbol{x})$ represents the expectation of random variable $\boldsymbol{x}$.

B. PRI Modulation

In order to eliminate distance ambiguity, velocity ambiguity, or eliminate target coverage in pulse Doppler radar, the PRI of radar is usually modulated. Common PRI modulation methods include the jitter PRI, the stagger PRI, the sliding PRI, and the group PRI. Denote the expectation of a PRI sequence as $\mu_{pri} = \frac{1}{T-1} \sum_{i=1}^{T-1} PRI(i)$. Together with the fixed PRI, five PRI modulation types are as follows:

$$PRI_{fixed}(i) = \mu_{pri} + \delta_F(i), \tag{4}$$

$$PRI_{jitter}(i) = \mu_{pri} + \delta_J(i), \tag{5}$$

$$PRI_{stagger}(i) = \mu_{pri} + A(i), \tag{6}$$

$$PRI_{sliding}(i) = \mu_{pri} + B(i), \tag{7}$$

$$PRI_{group}(i) = \mu_{pri} + C(i). \tag{8}$$

δ_F and δ_J are all the Gaussian distribution noise, but the standard deviation of δ_F is usually within 5% of the μ_{pri}. the standard deviation of δ_J is able to reach 30% of the μ_{pri}. Let β_A, β_B and β_C to be limitation factors. For the stagger PRI, given a PRI parameter set satisfying:

$$A = \left\{ A_\sigma \,\middle|\, |A_\sigma| \leq \beta_A, 0 \leq \sigma < K_A, \sum_{\sigma=0}^{K_A - 1} A_\sigma = 0 \right\}. \tag{9}$$

Let $mod(i, K_A) = \sigma$, and K_A means the size of the stagger parameter set. Then $A(i) = A_\sigma$. For the linear sliding PRI, we could define:

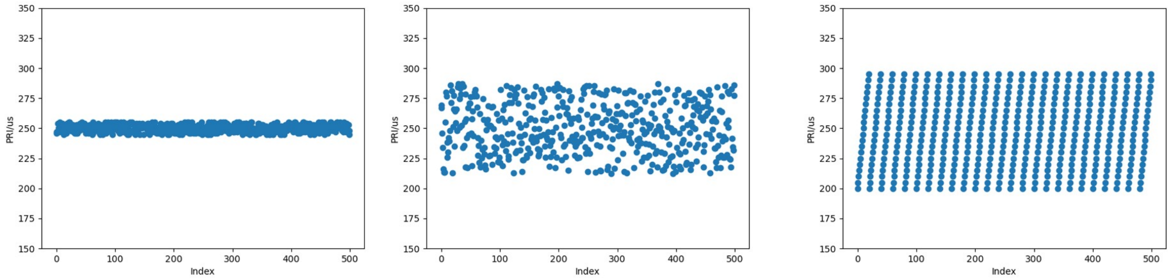
$$B(i) = \beta_B\left(\frac{\sigma}{K_B} - \frac{1}{2}\right)\mu_{pri}, \quad mod(i, K_B) = \sigma. \tag{10}$$

Similarly, if there are K_C PRI groups, then:

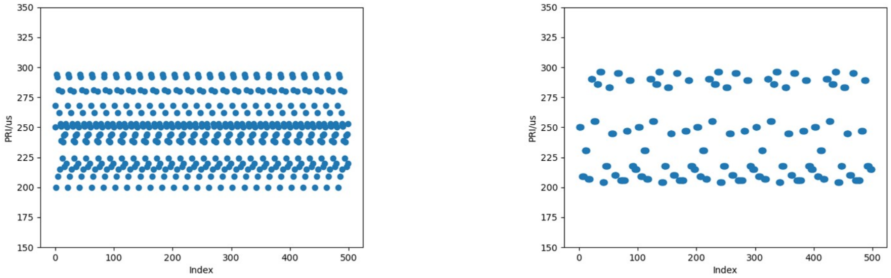
$$C = \left\{ C_\sigma \,\middle|\, |C_\sigma| \leq \beta_C, 0 \leq \sigma < K_C, \sum_{\sigma=0}^{K_C - 1} C_\sigma = 0 \right\}. \tag{11}$$

Each group has G points. When $\lfloor mod(i, GK_C)/G \rfloor = \sigma$, $C(i) = C_\sigma$. $mod(x/y)$ and $\lfloor x \rfloor$ represent take the remainder of x/y and round down x respectively.

Fig. 1 is a schematic diagram of the above five PRI modulation types. Depending on the radar system and function, the pulse sequence PRI modulation type is also different.



(a) Fixed PRI (b) Jitter PRI (c) Sliding PRI



(d) Stagger PRI (e) Group PRI

Fig. 1: PRI Modulation Types Schematic Diagram ($\mu_{pri} = 250\mu s$).

There are various PRI modulation types. Typical applications of fixed PRI are conventional search and track radar, pulse Doppler radar. Jitter PRI is used to counter the predicted ToA interference and reduce the specific interference effect. Stagger PRI is used to eliminate blind speed in MTI systems. Sliding PRI provides constant height coverage in pitch scans, avoiding shadowing effects. Group PRI is used to solve velocity or range ambiguities, especially for pulsed Doppler radars. [36].

Under the four conditions of unknown PRI, unknown number of radar emitters, environmental noise, and unknown modulation type, the problem of radar pulse deinterleaving is extremely difficult. We will propose three possible ways (with three different extra information) to solve it. But we mainly focuses on the deinterleaving problem when the first three constraints are met. We will prove in the simulation that as long as the modulation type of each emitter is known in advance, then the DTM algorithm can be extended regardless of the modulation parameter, even in the case of multi-mode modulation.

C. Non-Negative Matrix Factorization Deinterleaving

To separate the effective components from the mixed signal, common methods are spectral-based decomposition methods, such as Independent Component Analysis (ICA) and Non-negative Matrix Factorization (NMF). Since Lee and Seung [32] proposed the NMF algorithm, a series of NMF-based signal separation algorithms have been proposed. The process of Supervised NMF works as follows.

First, For short-time stationary signals, this feature extraction process is often done through Short-Time Fourier Transform (STFT). Assuming that the window function $w(t)$ is used to segment the signal, and the number of points in each frame is N, then the STFT is as follows:

$$\boldsymbol{X}(t,\omega) = \sum_{n=0}^{N-1} \boldsymbol{x}(n)w(t-n)e^{\frac{-2\pi j\omega n}{N}}. \tag{12}$$

However, the radar pulse train changes drastically in a real environment. Even in the case of the 1 μs time resolution selected in this paper, a short pulse segment is still not a stable signal. Moreover, the time-frequency resolution of STFT depends entirely on the length of the window function $w(t)$. If STFT is used for radar pulse deinterleaving, a very short window must be used, which results in low frequency resolution.

Next, perform NMF on the time-frequency spectrum $\boldsymbol{X}$, let $\boldsymbol{W}$ be the basis matrix and $\boldsymbol{H}$ be the coefficient matrix, decomposition is as follows:

$$\boldsymbol{X} = \boldsymbol{W}\boldsymbol{H}, \quad \text{s.t.} \quad \boldsymbol{W}, \boldsymbol{H} \geq 0. \tag{13}$$

In the $\boldsymbol{X}$ matrix, each column represents an observation and each row represents a feature; The $\boldsymbol{W}$ matrix is called the basis matrix, and the $\boldsymbol{H}$ matrix is called the coefficient matrix or the weight matrix. At this time, using the coefficient matrix $\boldsymbol{H}$ instead of the original matrix can reduce the dimension of the original matrix and obtain the dimension reduction matrix of data features, so as to reduce the storage space. The basic principle of NMF blind signal separation is to decompose the signal into a non-negative basis matrix and the corresponding coefficient matrix. Then according to the coefficient mask corresponding to each emitter, each signal source is finally sorted.

In the training stage, the base matrix $\boldsymbol{W_k}$ of each radar pulse sequence is trained based on the supervision knowledge. In the test stage, $\boldsymbol{W_k}$ is fixed, and only the coefficient matrix

This article has been accepted for publication in IEEE Transactions on Aerospace and Electronic Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TAES.2022.3193948

H_k is calculated. Let $\odot$ denotes the Hadamard product, the estimated time-frequency features X_k of each radar pulse train are obtained:

$$X = \sum_{k=1}^{K} X_k = \sum_{k=1}^{K} W_k H_k \tag{14}$$

$$X_k = \frac{W_k H_k}{WH} \odot X. \tag{15}$$

In addition, there are many improved NMF methods, such as the Projective NMF (PNMF) [37] algorithm proposed by combining Principal Component Analysis (PCA) and NMF, the Sparse NMF method [38] that further increases the sparsity constraint, and so on. However, these methods are not suitable for radar pulse deinterleaving using STFT transformation. They still need to iteratively compute the sparse matrix H_k during testing. It is not accurate and robust enough for complex deinterleaving problem. In order to perform non-negative matrix factorization, the signal must first be projected into a sparse feature space. Fourier transform performs time-frequency conversion from the signal as a whole. After processing non-stationary signals, time information is lost in the spectrum, which makes it impossible to sort radar pulse train.

III. Proposed Method

In the case of the fixed PRI, although the accuracy is not high, radar pulse sequences of a small number of emitters can be deinterleaved using sequence search like methods. When the number of radar emitters is known, the cluster based algorithm can also deinterleave the radar sequence of unknown PRI. However, for unknown PRI, noisy jitter PRI pulse sequences of unknown radar emitter number, even the existing deep learning methods cannot solve it. Not to mention the sorting of interleaved pulse sequences in the case of multiple unknown PRI modulation modes. But the DTM algorithm proposed in this paper could solve it under certain conditions.

A. Radar Pulse Deinterleaving Framework

We propose an algorithm, namely the DTM algorithm, which is inspired by Supervised NMF. In EW, the interleaved radar pulse signals are usually streaming data continuously arriving at the receivers. They cannot be processed at once. The whole deinterleaving processing framework is shown in Fig. 2. After formalizing the ToA train S of the received PDW data D as $x(n)$ according to Section II, all available PDW parameters except ToA are saved as a extra feature vector sequence V for optional post-processing algorithms, i.e., $D = [V; S]$. The interleaved radar pulse trains x and those separated pulse trains x_k can be obtained by the simulation data or by collecting the offline data. We segment the long pulse sequence into M frames. Each frame contains N pulse sampling point data, $s = [x_i, x_{i+1}, ..., x_{i+N-1}]$, $i = 1, 2, ..., M$. s_k indicates a pulse frame of the radar emitter k. y_s is the emitter labels of s, and the value of s_k could be inferred easily through it. These frames form a processed pulse data set $\{s^{(j)}, y_s^{(j)}\}$, $j = 1, 2, ...J$, including a total of J pairs of data. Input each frame of pulse data s into the RDN model for offline training according to our designed loss function. After the training stage is completed, we perform online pulse deinterleaving to evaluate the DTM algorithm. We could label some separated radar pulse trains to fine tune the model offline. On the other hand, it is possible to fine tune the RDN online for pre-processing. The purpose is to adaptively adjust the RDN parameters. At the same time, the RDN predicts the number of radar emitters and sorts a frame of radar pulse train based on the ToA mask. The ToA mask is a coefficient matrix of the corresponding radar emitter in the ToA feature space. It represents the weight, or the proportion of each individual radar ToA feature to the interleaved radar ToA feature on the basis of the feature space. After it, we use the other parameters V as the input of post-processing to perform re-clustering and get more robust results. The results are used as the dataset for the online fine-tuning. The network is dynamically adjusted in the background to avoid affecting the real-time performance of the online sorting. We update the RDN parameters after the fine-tuning is completed. Repeat the above process for subsequent radar pulse frames until all the frames are processed. At last, label which radar emitter each PDW point in the train belongs to, then the sorting is finished. The DTM algorithm includes the RDN training stage and the whole online radar deinterleaving process.

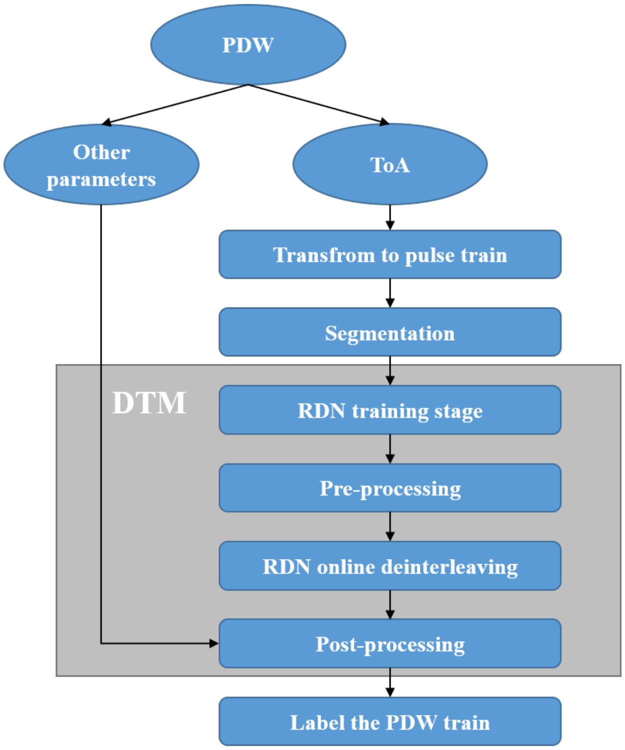


Fig. 2: Radar pulse deinterleaving processing flow.

B. Recursive Deinterleaving Network

The key part of the DTM is the RDN, which is shown in Fig. 3. It is a network that combines dilated convolution blocks with intra-frame and inter-frame attention. This structure is conducive to integrate feature information and extract pulse sequence patterns. The second column of Table I shows the

This article has been accepted for publication in IEEE Transactions on Aerospace and Electronic Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TAES.2022.3193948

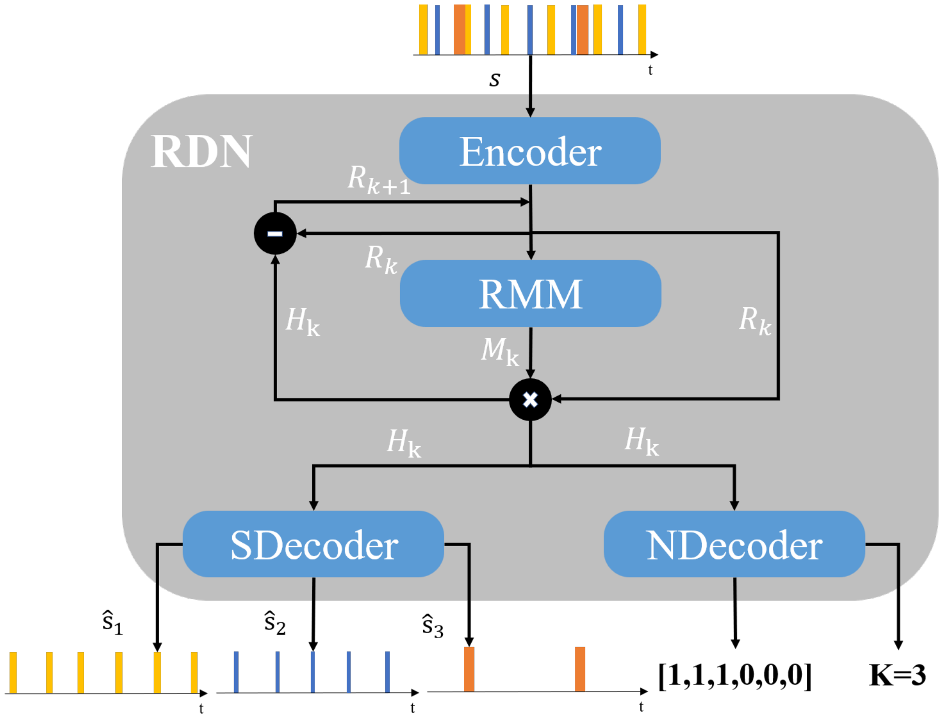


Fig. 3: Structure of the recursive deinterleaving network.

TABLE I: Input shapes of the RDN modules

Module Name	Shape	Specific shape
Encoder	(B, T)	$(4, 60000)$
RMM	(B, F, L)	$(4, 256, 3749)$
Decoder	(B, K, F, L)	$(4, 6, 256, 3749)$

shape of the input data for each module of the RDN, and the third column shows the specific input shape of each module of the RDN during simulation. The first dimension B represents the batch size of the training data. The output of the Decoder is the same shape as the input of the Encoder. The RDN has four main differences from Supervised NMF.

First, the RDN includes a learnable and adaptive convolutional encoder to map the radar pulse frame $\boldsymbol{s}$ to the coefficient matrix $\boldsymbol{H}$ directly from the time domain. It implicitly encodes radar pulse sequences to an appropriate feature space rather than transforms it to the time-frequency domain by STFT. The decoder convolution kernels are similar to the basis matrix $\boldsymbol{W}_k$. But the decoder is used to map the deinterleaved pulse feature to the pulse label sequence $\boldsymbol{y}_s$ rather than the STFT spectrum $\boldsymbol{X}_k$. Second, the ToA mask of each emitter is calculated with the pulse context information by dilated convolutions. Dilated convolution refers to the convolution with additional holes in its kernel. Compared with the normal convolution, the dilated convolution has one more hyper parameter, called the dilation rate, which refers to the step size between two values in the kernel (for the normal revolution, the dilation rate is 1) [39]. PRI short-term mode and long-term mode information is also merged into the ToA mask through dual-path attention. In other words, the RDN does not simply and directly estimate the coefficient matrix $\boldsymbol{H}_k$ of each emitter like NMF. Third, the process of the RDN to compute the ToA coefficient mask is recursive. At each iteration, a certain radar ToA mask $\boldsymbol{M}_k$ of the emitter k is extracted from the residual ToA coefficient matrix $\boldsymbol{R}_k$ through the Recursive Mask Module (RMM). Then the residual interleaved pulse ToA coefficient matrix $\boldsymbol{R}_{k+1}$ filtered by the ToA mask $\boldsymbol{M}_k$ are sent to the next iteration to estimate the next ToA mask $\boldsymbol{M}_{k+1}$. That is, in each iteration, the coefficient matrix $\boldsymbol{H}_k$ are removed from $\boldsymbol{H}$ to simplify the problem. Fourth, the RDN also automatically predicts the number of radar emitters based on the deinterleaved radar feature.

The encoder layer includes convolution kernels $\boldsymbol{B}$ and ReLU activation function. It ensures that the coefficient matrix $\boldsymbol{H}$ of the Encoder is non-negative. As mentioned earlier, the radar pulse sequence is a non-stationary signal even in a short period of time. So it is difficult to model a radar pulse train with linear functions. The nonlinear activation makes the representation ability of features stronger. The Encoder is as follows:

$$\boldsymbol{H} = Encoder(\boldsymbol{B}, \boldsymbol{s}) = ReLU(Conv(\boldsymbol{B}, \boldsymbol{s})), \boldsymbol{H} \in \mathcal{R}^{F \times L}. \tag{16}$$

where F is the feature dimension of $\boldsymbol{H}$, and L is the time dimension. Here, if $\boldsymbol{H}$ is decoded, the reconstructed interleaved pulse frame $\hat{\boldsymbol{s}}$ can be obtained. By decoding the $\boldsymbol{H}_{\boldsymbol{k}}$ obtained by RMM, we will get the deinterleaved single pulse frame $\hat{\boldsymbol{s}}_{\boldsymbol{k}}$:

$$P^{(s)} = Sigmoid(DeConv(\boldsymbol{W}, \boldsymbol{H})) \tag{17}$$

$$P_k^{(s)} = Sigmoid(DeConv(\boldsymbol{W}, \boldsymbol{H}_{\boldsymbol{k}})) \tag{18}$$

$$\hat{\boldsymbol{s}} = SDecoder(\boldsymbol{W}, \boldsymbol{H}) = I(P^{(s)} > t_s) \tag{19}$$

$$\hat{\boldsymbol{s}}_{\boldsymbol{k}} = SDecoder(\boldsymbol{W}, \boldsymbol{H}_{\boldsymbol{k}}) = I(P_k^{(s)} > t_s). \tag{20}$$

This article has been accepted for publication in IEEE Transactions on Aerospace and Electronic Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TAES.2022.3193948

Here, $Sigmoid(x) = \frac{1}{1+e^{-x}}$, which is an activation function. $DeConv$ denotes deconvolution layer.

The SDecoder could be implemented by either a fully connected layer or a deconvolution layer. The realization of deconvolution is more efficient. The radar pulse point is always a 0-1 binary variable, so the confidence $P_k^{(s)}$ will be estimated through the Sigmoid function at the end of the SDecoder. $I(x)$ is an indicator function. When the confidence that a pulse frame belongs to the k-th radar is greater than the threshold t_s, the indicator function $I(P_k^{(s)} > t_s) = 1$; otherwise, $I(P_k^{(s)} > t_s) = 0$.

It is not suitable to select Softmax for normalization here, because the radar pulse frames may overlap. At this time, a certain pulse frame might belong to multiple radars. The radar pulse trains emitted by different emitters have a weak correlation. This feature is a basis for judging whether the pulse trains of different modes belong to the same radar emitter. Due to the existence of noise in the realistic warfare environment, there is often a certain gap between the reconstructed $\hat{\boldsymbol{s}}$ and $\boldsymbol{s}$. Significantly, there is an assumption in the DTM that the basis matrix $\boldsymbol{W_k} = \boldsymbol{W}$. There are enough neural network parameters, and this simplification can reduce its complexity.

The main part of RMM are two deep separable dilated convolution block and the dual-path attention block. It is shown in Fig. 4. It decomposes $\boldsymbol{H}$ after fusing local and global information. Local information refers to the local relationship of radar pulse points, which is reflected in the change of radar pulse sequence in a short time (usually within 2000 μs in simulation). Global information refers to the mode presented by the whole pulse frame or even the whole pulse sequence. This mode is more macro (about 60000 μs are usually considered in simulation). The reason for this consideration is due to the existence of multi-mode radar pulse modulation. Taking the group PRI as an example, the PRI changes within groups are local information, while PRI changes between groups are global information. Through this module, the RDN iteratively predicts the ToA mask $\boldsymbol{M_k}$ from the coefficient matrix, and obtains $\boldsymbol{H_k}$. Bottleneck refers to a neural network structure in which the dimension of the front and back layers of neurons is higher than that of the middle layer. This structure looks like a bottleneck. However, our BottleNeck here only includes the first two layers, which mainly plays the role of dimension reduction. After Bottleneck, the feature dimension F is reduced to F'. The activation Parametric Rectified Linear Unit (PReLU) is defined as:

$$PReLU(y_i) = \begin{cases} y_i, & y_i > 0 \\ \gamma_i y_i, & y_i \leq 0. \end{cases} \tag{21}$$

where γ_i is a learnable parameter, and is the negative slope of the activation function. y_i is the input of nonlinear activation function in the i-th channel.

Assuming that $\boldsymbol{R_k}$ is the residual coefficient matrix before iteration k, and $\boldsymbol{R_1} = \boldsymbol{H}$ in the first iteration, then the recursive deinterleaving is described as follows:

$$\boldsymbol{M_k} = RMM(\boldsymbol{R_k}) \tag{22}$$

$$\boldsymbol{H_k} = \boldsymbol{M_k} \odot \boldsymbol{R_k} \tag{23}$$

$$\boldsymbol{R_{k+1}} = \boldsymbol{R_k} - \boldsymbol{H_k}, \tag{24}$$

where $k = 1, 2, ..., K_{max}$, and K_{max} is the maximum possible number of radar emitters. In RMM, dilated convolution layers with dilation rate of 2^b are used, where $b = 1, 2, ..., B'$. Receptive field is defined as the size of the input area after the pixels on the output feature map of each layer are mapped to the input. It is the size of a point on the output feature map relative to the input feature of the network. The radar pulse train changes rapidly, and the discrimination of radar modulation mode requires long-term information. Therefore, the receptive field must be expanded as much as possible while taking the time resolution into account. The dilated convolution layer reduces the amount of parameters with deep separable convolution structure. Deep separable convolution decomposes the ordinary convolution into two series operations, i.e., point-by-point convolution and channel convolution, and improves the inference calculation efficiency of the network. For the pulse sequence down-sampled by Encoder in 16 times, each deep separable dilated convolution block contained $B' = 11$ separable convolution structures. The receptive field of each block is 8191. Therefore, it is enough to get the pulse PRI characteristics.

We divide the features processed by the first deep separable dilated convolution block into S frames, denoted as $\boldsymbol{Z} \in \mathcal{R}^{F' \times S \times L'}$. Then the neighborhood patterns of the radar pulse sequence are encoded in each frame internally. And its global patterns are encoded between the frames. Attention essentially assigns a weight to the feature vector at each time step in the feature matrix $\boldsymbol{Z}$. If this weight is a one-hot vector, then a context vector that is most relevant to the current vector will be selected. At this time, this one-hot vector can be regarded as an address on the time step, which is called hard addressing. However, the one-hot vector is not differentiable and it needs to be solved with the help of the policy gradient. The weight coefficients in the RMM are continuous values, which can be understood as soft addressing. If each element in a sequence is stored in the form of $(\boldsymbol{K}, \boldsymbol{V})$, then attention completes the addressing by calculating the similarity between $\boldsymbol{Q}$ and $\boldsymbol{K}$. The similarity calculated by $\boldsymbol{Q}$ and $\boldsymbol{K}$ reflects the importance of the extracted $\boldsymbol{V}$. We first model the correlation within the frame through intra-frame attention, and then model the correlation between the frames through inter-frame attention. This dual-path attention mechanism is good for the recognition of radar modulation. Taking group PRI as an example, intra-frame attention can extract constant PRI information in each group, and inter-frame attention is responsible for analyzing the changes in PRI between groups. The intra-frame attention is as follows:

$$\boldsymbol{K_1}, \boldsymbol{Q_1}, \boldsymbol{V_1} = Linear(\boldsymbol{Z}) \tag{25}$$

$$IntraAttention(\boldsymbol{K_1}, \boldsymbol{Q_1}, \boldsymbol{V_1}) = Softmax\left(\frac{\boldsymbol{Q_1}\boldsymbol{K_1^T}}{\sqrt{F'S}}\right)\boldsymbol{V_1}. \tag{26}$$

This article has been accepted for publication in IEEE Transactions on Aerospace and Electronic Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TAES.2022.3193948

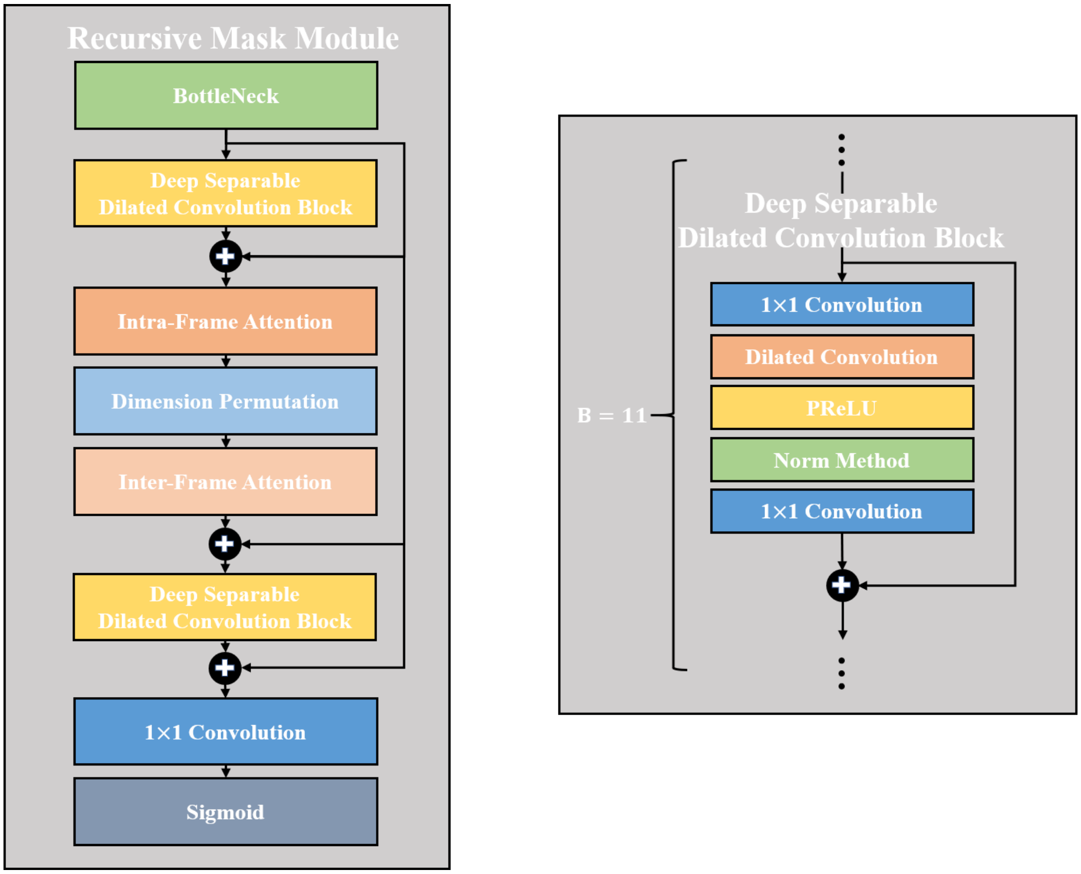


Fig. 4: Structure of Recursive Mask Module (left); Details of dilated convolution layer (right).

Similarly, after dimension permutation, for $\boldsymbol{Z}^T \in \mathcal{R}^{F' \times L' \times S}$, inter-frame attention is as follows:

$$\boldsymbol{K_2}, \boldsymbol{Q_2}, \boldsymbol{V_2} = Linear(\boldsymbol{Z}^T) \tag{27}$$

$$InterAttention(\boldsymbol{K_2}, \boldsymbol{Q_2}, \boldsymbol{V_2}) = Softmax\left(\frac{\boldsymbol{Q_2} \boldsymbol{K_2^T}}{\sqrt{F'L'}}\right)\boldsymbol{V_2}. \tag{28}$$

Before dimension permutation, we use intra-frame attention to get local information, which is equivalent to stacking adjacent ToA pulse points together as features. Dimension permutation is to stack ToA pulse points with a certain interval as features for inter-frame attention and to get global information.

The normalization method and the residual connection are important guarantees for the good results of the RMM. The LayerNorm method [40] simultaneously normalizes the coefficient matrix in the two dimensions, feature and time. The MeanNorm method proposed in this paper is based on the GloabalNorm. Moreover, it re-parameterizes the mean and standard deviation of the normalized coefficient matrix $\boldsymbol{H_k}$ based on sorting results got from the first deep separable dilated convolution block and dual-path attention. Therefore, the normalization method in the first deep separable dilated convolution block is LayerNorm, but in the second one is MeanNorm. In RMM, we have added a lot of residual connections between layers and between blocks to prevent the gradient vanishing problem.

In fact, the RMM computes the ToA mask $\boldsymbol{M_k}$ from the pulse sequence feature, then separates the coefficient matrix of the k-th pulse train from the remaining coefficient matrix $\boldsymbol{R_k}$. Since $\boldsymbol{W} = \boldsymbol{W}_k$, whether $\boldsymbol{W}$ is a non-negative matrix or not, it is equivalent to the iterative process of Supervised NMF as follows:

$$\boldsymbol{M_k} \prod_{i=1}^{k-1}(1 - \boldsymbol{M_i}) = \frac{\boldsymbol{W_k H_k}}{\boldsymbol{WH}} = \frac{\boldsymbol{H_k}}{\boldsymbol{H}}. \tag{29}$$

$\boldsymbol{W}_k$ can be constrained to be non-negative, but we find that not specifically constraining it works better. In fact, whether $\boldsymbol{W}_k$ is non-negative does not affect the RMM derived by NMF, but only affects what kind of latent space the Encoder-Decoder maps the radar signal to.

Most of the current radar pulse deinterleaving algorithms cannot determine the number of radar emitters in a noisy environment well. The RDN directly predicts the existence probability of a radar pulse train based on the coefficient matrix:

$$P_k^{(n)} = NDecoder(\boldsymbol{H_k}) = Sigmoid(Linear(\boldsymbol{H_k})) \tag{30}$$

$$K = \sum_{k=1}^{K_{max}} I(P_k^{(n)} > t_n). \tag{31}$$

$Linear(.)$ denotes linear transformation. Similar to SDecoder, NDecoder is a decoder for predicting the number of radar emitters, and t_n is the confidence threshold for the existence of a radar emitter. After the NDecoder branch, the RDN specifically predicts if a radar emitter exists or not. By summing the existence probability up, the emitter number is easy to get.

C. DTM Loss Function

For short-time stationary speech signals, the permutation invariant signal-to-noise ratio improvement rate is generally used as the loss function. However, the biggest difference between the decoded pulse frame $\hat{\boldsymbol{s}}_{\boldsymbol{k}}$ and the ground-truth $\boldsymbol{s}_{\boldsymbol{k}}$ is that $\boldsymbol{s}_{\boldsymbol{k}}$ is highly sparse, but $\hat{\boldsymbol{s}}_{\boldsymbol{k}}$ does not have sparseness before rounding from probability sequence to 0-1 sequence according to the threshold t_s. A small value change near t_s in $P_k^{(s)}$ during training stage will cause $\hat{\boldsymbol{s}}_{\boldsymbol{k}}$ to change from 0 to 1, or from 1 to 0. This will cause severe fluctuations in the calculation of the signal-to-noise ratio and difficulty in parameters convergence. Furthermore, the threshold t_s is not easy to estimate during the training process. Therefore, a loss function based on the signal-to-noise ratio cannot be used for discrete pulse signals. The cross-entropy loss function defined by the pulse distribution distance would be a better choice.

The channel permutation problem means that any permutation of the radar pulse source on different output channels is equivalent, resulting in that the same pulse source may be allocated to different channels. In the deinterleaving process, we need to define the sequence relationship for the radar pulse source to avoid channel permutation problems. The value of the ToA mask $\boldsymbol{M}_{\boldsymbol{k}}$ belongs to $(0, 1)$. If it is specified that the norm of the radar coefficient matrix $\boldsymbol{H}_{\boldsymbol{k}}$ is arranged in descending order of the iteration rounds, i.e., $||\boldsymbol{H}_{\boldsymbol{k}}|| \leq ||\boldsymbol{H}_{\boldsymbol{k-1}}||$, then the relationship of the output channels is determined. Under the output channel arrangement rule mentioned above, the expectation of PRI μ_{pri} will increase as the radar emitter identification k increases, i.e., $\mu_{pri,k} \geq \mu_{pri,k-1}$, which is more natural. In other words, as the radar emitter ID k increases, the number of pulse points in the sequence will decrease.

This approach can be ambiguous when multiple radar emitters have similar PRI μ_{pri}, or when the radar pulse sequences are influenced by PRI modulation modes and environmental noise. This is because the feature norms $||\boldsymbol{H}_{\boldsymbol{k}}||$ and $||\boldsymbol{H}_{\boldsymbol{k-1}}||$ may be very similar at this time. Another approach combines the advantages of the RDN's recursive deinterleaving property. After sorting out the pulse sequence $\hat{s}_k$ in each iteration, it is possible to calculate which real radar pulse sequence s_k is closest to $\hat{s}_k$, and assign $\hat{s}_k$ to the corresponding radar emitter k. Then we compute the deinterleaving loss between the two sequences. The total loss is the sum of the losses of each channel $\hat{s}_k$. In the test stage, this training strategy ensures that each channel of the deinterleaved pulse sequence only includes the pulse points of a certain radar emitter. There can be a certain overlap between the radar pulse frames. Through the overlap between different frame, we can compute the similarity of the overlap part between the current deinterleaved radar pulse frame and the previous deinterleaved frame, thereby ensuring that current channel assignments are consistent with the previous ones. It ensures the sequential channel consistency. In this way, the deinterleaving accuracy will not be affected by the ambiguity of the feature norm $||\boldsymbol{H}_{\boldsymbol{k}}||$. Since similar PRI μ_{pri} of two sequences does not exist in our simulation, we use the first simpler method to solve the channel permutation problem. But the second method would be more general.

Based on the above analysis, we design a special cross-entropy loss function to measure the distance between the predicted pulse distribution and the ground-truth distribution. We only care about whether $\hat{\boldsymbol{s}}_{\boldsymbol{k}}(n) = \boldsymbol{s}_{\boldsymbol{k}}(n)$ holds when $\boldsymbol{s}(n) = 1$, and do not make further requirements for the performance of the DTM algorithm when $\boldsymbol{s}(n) = 0$. Because of the existence of noise, such as PRI jitter or pulse missing, $\hat{\boldsymbol{s}}(n) > 0$ often holds when $\boldsymbol{s}(n) = 0$. The stronger the noise in the environment, the higher the probability of pulse points occurrence. If the probability distribution $P_k^{(s)}$ is relatively flat, it means the probability of the pulse frame $\boldsymbol{s}_k(n) = 1$ is allocated to the adjacent time point. Therefore, it is not appropriate to constrain all radar pulse frames to satisfy $\hat{\boldsymbol{s}}_{\boldsymbol{k}}(n) = 0$ when $\boldsymbol{s}(n) = 0$. Let $\boldsymbol{ps}_{\boldsymbol{k}} = \{n|\boldsymbol{s}(n) = 1 \wedge \boldsymbol{s}_{\boldsymbol{k}}(n) = 1\}$, and $\boldsymbol{ns}_{\boldsymbol{k}} = \{n|\boldsymbol{s}(n) = 1 \wedge \boldsymbol{s}_{\boldsymbol{k}}(n) = 0\}$ respectively represent the positive sample set and the negative sample set, then the deinterleaving cross-entropy loss is as follows:

$$L_D = -\sum_{k=1}^{K}\Big[\sum_{n\in ps_k} \boldsymbol{s}_{\boldsymbol{k}}(n)\log P_k^{(s)}(n) + \sum_{n\in ns_k}(1-\boldsymbol{s}_{\boldsymbol{k}}(n))(1-\log P_k^{(s)}(n))\Big]. \tag{32}$$

The cross-entropy loss of radar existence is as follows:

$$L_N = -\sum_{k=1}^{K}\log P_k^{(n)} - \sum_{k=K+1}^{K_{max}}\log(1 - P_k^{(n)}). \tag{33}$$

In other words, we set it as default that when a radar pulse train exists, it will always be assigned to the foremost channel first. Only when $K = K_{max}$, the last channel represents a radar pulse probability sequence. In summary, the total loss is as follows:

$$\mathbf{L} = L_D + \lambda L_N. \tag{34}$$

λ is the weighting factor that weighs the two loss functions L_D and L_N.

D. Pre-Processing and Post-Processing for Deinterleaving

After training is over, the RDN can be directly applied in the realistic EW environment and sort radar pulse trains of multiple PRI modulations accurately. However, without prior knowledge of the complex environment and the PDW information except for the ToA, the DTM algorithm is not robust enough.

Radar emitters often emit pulse sequence continuously, and it changes rapidly. Calculated with the time resolution of 1 μs in this paper, 10^6 pulse sample points can be generated within 1s. Specific to a certain complex environment, the processor of the radar receiver may have certain prior knowledge of it, such as radar emitter type, PRI modulation, noise estimation, and so on. Before application, it is possible to fine tune the RDN based on prior knowledge, either by constructing simulation data or by using the radar pulse data collected in the past. Even training the model from scratch requires only about pulse sequence data of about ten minutes. A few seconds of data is enough to fine tune the network parameters. Specifically, after the receiver receives the radar pulse sequence, the RDN performs pre-sorting for a frame of the sequence first. These radar pulse frames and corresponding deinterleaving results can be used to fine tune the RDN offline or online.

To fine tune the model to adapt to the environment, we could collect some deinterleaving radar sequences, and simply process them by removing abnormal points and making corrections manually. After that, they are available to fine tune the RDN offline. During online sorting, if we want to fine tune the RDN, we should ensure the accuracy of the pre-sorting results. By using Sequence Search and some post-processing methods, we could obtain more accurate results. The DTM takes advantage of the accurate data for fine-tuning training in the backend. After a period of training time, we replace the current model with the updated model. Online sorting and pre-processing do not interfere with each other. This method combines low-frequency model update with high-frequency pulse deinterleaving, which not only performs pulse deinterleaving in real time, but also ensures the accuracy of the result. The warm-up of the LSTM-based method requires re-training the LSTM model with a certain amount of supervised data before use, otherwise the effect will be very unstable. Relatively speaking, it is closer to offline fine-tuning, and this is a necessary step for the LSTM-based method. The online fine-tuning used in DTM algorithm in our paper does not need to obtain supervised data by manually labeling pulse sequences, but automatically labels them in the background through DTM algorithm. The reason why DTM algorithm can do this is mainly due to its high precision. Even without preprocessing, it can accurately deinterleave the pulse train, which is not the case with LSTM-based methods.

On the other hand, there are often many post-processing methods that could be used to further improve the accuracy of the DTM. The tracking-based algorithm mentioned above is one of them. In addition, the cluster based method is usually a good choice. The PDW vector $\boldsymbol{V}$ could be utilized as the input of the re-clustering algorithm. The key question of the post-processing is how to better integrate the RDN pre-sorting results with the feature $\boldsymbol{V}$. A feasible method is to use the believable pre-sorting results as the emitter label to initialize the cluster center. Moreover, the probability sequence $P_k^{(s)}$ predicted by the RDN is able to use as weights of sample points when calculating the feature similarity. The Gaussian Mixture Clustering (GMM) algorithm makes use of it as the estimated posterior probability. The RDN also predicts the number of radar emitters, and the prediction is very accurate. With this information, the number of clusters is determined in the post-processing. Therefore, the accuracy of the re-clustering post-processing is greatly improved compared with performing clustering directly.

E. Processing of Different PRI Modulation Modes

In fact, only relying on ToA to deinterleave radar pulse trains with complex modulation patterns is problematic. If neither the number of PRI and radar emitters nor the modulation mode is known, the ToA information alone is easy to cause ambiguity when deinterleaving. Among them, stagger PRI is the most typical. The deinterleaving algorithm using only ToA is very likely to treat each component $\mu_{pri} + A_k$ in stagger PRI as an independent radar emitter. As shown in Fig. 5, the same radar pulse sequence can be interpreted as a single radar emitter sequence of stagger PRI or a three emitters sequence of fixed PRI. In this example, we can analyze that if it is a fixed PRI sequence, it is unreasonable that the PRI of each emitter is identical. Once the environmental missing rate rises and multiple modulated radar pulse sequences appear at the same time, can we still distinguish and deinterleaving? The dual-path attention can alleviate this problem to a certain extent by analyzing inter-frame information. However, in order to reduce the amount of network parameters and the required training samples to make the DTM algorithm more efficient, it would be better if other information can be introduced to help reduce ambiguity.

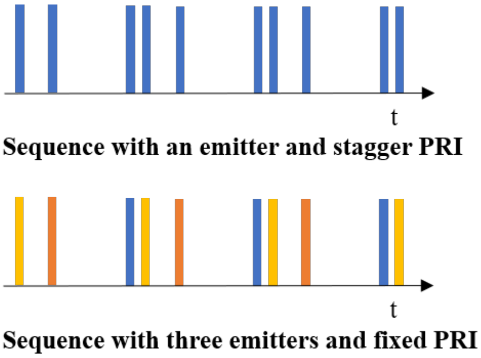


Fig. 5: An example of the stagger PRI ambiguity.

We propose three possible methods to make the DTM algorithm better applied to the deinterleaving of radar pulse trains with multiple modulation modes. The first method is to introduce PRI information to resolve ambiguity. But it is difficult to know the specific pulse PRI in advance. The second method is introduce PRI modulation type information, it may be known in advance relative to PRI information. It should be noted that the DTM algorithm only needs to know the modulation type of the PRI, and does not need to be specific to the modulation parameters. We can also add a PRI modulation type prediction module before or in the RDN to realize the prediction through the PRI long-term relationship, and then guide the radar pulse deinterleaving through it. It is reserved for our future work. The third method is to treat each component of stagger PRI and group PRI as a single PRI. After deinterleaving by the RDN, the noisy PDW information is used to assist in determining which emitter these components belong to. This method ignores the relationship between the pulse groups and considers that the radar pulse trains of different PRIs are independent of each other. Therefore, after the RDN, the deinterleaving results must be re-clustered based on the pulse correlation of the same radar emitters.

IV. SIMULATION

A. Datasets

There is no standard dataset for the radar pulse deinterleaving task, so the data used in this exiperiment are all noisy PDW generated by simulation. The parameters of the simulation are shown in Table II. The dataset is divided into a training dataset, validation dataset, and test dataset. Each dataset is

This article has been accepted for publication in IEEE Transactions on Aerospace and Electronic Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TAES.2022.3193948

TABLE II: Information of radar parameters

Parameters	Value range
Time Resolution	$1\mu s$
Minimal Delta	$20\mu s$
TOA	$1\mu s - 60000\mu s$
PRI	$100\mu s - 600\mu s$
PW	$1\mu s - 5\mu s$
DOA	$30° - 90°$
RF	5.0MHZ-15.0MHZ
Emitter Number (K)	1-6
Jitter Rate (θ_J)	0-0.3
Missing Rate (θ_M)	0-0.3
Stagger PRI Set Size (K_A)	2-7
Stagger Limitation Factor (β_A)	50
Sliding PRI Set Size (K_B)	4-10
Sliding Limitation Factor (β_B)	100
Group PRI Set Size (K_C)	2-5
Group Limitation Factor (β_C)	100
Group PRI Number (G)	2-8

independently generated by a different random seed. A frame of pulse train $\boldsymbol{s}$ contains 60 ms of PDW, and the ToA train will be converted into a pulse form. The pulse sequence always starts from 1 and ends at 60000. A frame of data sample includes more than one individual pulse frame $\boldsymbol{s_k}$ as labels, and the mixed pulse frame $\boldsymbol{s}$ is used as the data to be sorted. There are some pulse sequences emitted by a single radar emitter in the training dataset to help the network learn the separated radar feature. The emitter radar parameter or the means of them are randomly selected from Table II. The mean of PRI is μ_{pri}, and the standard deviation is $\delta_{pri} = 0.05\mu_{pri}$. The mean of PW is μ_{pw}, and the standard deviation is $\delta_{pw} = 0.5\mu_{pw}$. Because the time resolution of the model is only 1 μs, both PRI and PW are rounded up. The first pulse point of a sequence is randomly selected from 1 to μ_{pri}. The minimum PRI difference of each radar sequence constituting the interleaved pulse sequence is determined by the parameter ”Minimum Delta”, which is $20\mu s$ in our simulation.

We first generated a jitter PRI modulated radar pulse dataset. The training dataset includes a total of 8000 pulse sequences with random radar parameters, and the total duration is 8 minutes with a time resolution of 1 μs. The validation dataset and the test dataset include independently generated 4000 sequences. And the interleaving pulse sequence is mixed by random from 2 to 6 generated single radar pulse sequences. To simulate dynamically changing signals in a complex EW environment, different degrees of Gaussian distributed noise will be added according to the jitter rate θ_J and the missing rate θ_M during the generation process. θ_J causes the PRI of radar pulse trains to vary randomly within δ_J, and $\delta_J = \theta_J\mu_{pri}$. θ_M causes a certain percentage of radar pulse trains to be lost.

Then we generate a dataset with multiple PRI modulations and a variable number of radar emitters. PRI modulation methods include jitter PRI, stagger PRI, sliding PRI and group PRI. The radar modulation parameters are shown in Table II, and the meaning of each parameter can be found in Section II-B. The PRI modulation modes and parameters are randomly selected within a range to better deal with different types and working modes of radar in real environments rather than a certain fixed situation. Except for the change of PRI modulation mode, other settings are similar to the jitter PRI dataset.

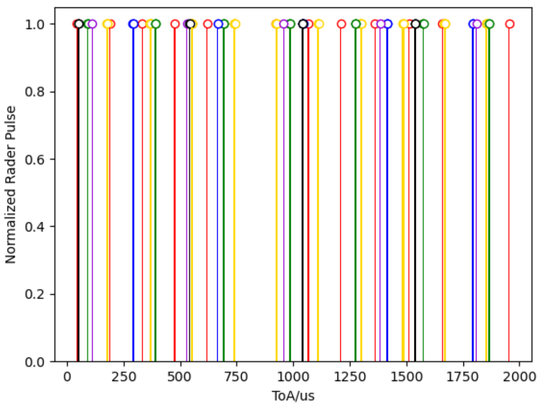


Fig. 6: Interleaving radar pulse train of six radars.

As shown in Fig. 6, it is an interleaved radar pulse frame generated by six radar emitters with $\theta_J = \theta_M = 0.1$. It can be found that although a single radar pulse sequence is sparse, the mixed sequence is relatively dense. In about $ToA = 50\mu s$, multiple radar pulse trains have overlapped. Near $ToA = 850\mu s$, the purple radar is obviously missing. It can be seen that when the number of radars is larger, the deinterleaving task becomes more difficult. Affected by the noise in the environment, the traditional deinterleaving algorithm will be difficult to play a role.

B. *Evaluation Criteria*

The formal radar pulse deinterleaving task mainly measures whether the pulse points belonging to each emitter is predicted correctly. It includes two aspects of evaluation. The first is the most straightforward criterion for evaluating the deinterleaving algorithm: evaluating the prediction accuracy of each sample point ToA_i in the ToA train. For the prediction $g : ToA \to y$ from the ToA train to emitter label, prediction accuracy is as follows:

$$Accuracy = \frac{1}{T}\sum_{i=1}^{T} I(g(ToA_i) = y_i). \tag{35}$$

However, the DTM algorithm directly processes the radar pulse train $\boldsymbol{x}$ instead of the ToA train, so the ToA accuracy cannot intuitively reflect the performance of the algorithm. Referring to the loss function, we use the precision and recall of the multi-classification problem to evaluate it. There are four related indicators: True Positive (TP), True Negative (TN), False Positive (FP) and False Negative (FN). TP means the number of elements in the set $\{n|\boldsymbol{s}(n) = 1 \wedge \boldsymbol{s_k}(n) = 1 \wedge \hat{\boldsymbol{s}}_{\boldsymbol{k}}(n) = 1\}$, and TN refers to the number of elements in the set $\{n|\boldsymbol{s}(n) = 1 \wedge \boldsymbol{s_k}(n) = 0 \wedge \hat{\boldsymbol{s}}_{\boldsymbol{k}}(n) = 0\}$. The definitions of FP and FN are similar to the previous two. Then the precision P

This article has been accepted for publication in IEEE Transactions on Aerospace and Electronic Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TAES.2022.3193948

and recall R are as follows:

$$P = \frac{TP}{TP+FP} \tag{36}$$

$$R = \frac{TP}{TP+FN}. \tag{37}$$

F1 is the harmonic average of precision and recall,

$$F1 = \frac{2PR}{P+R}. \tag{38}$$

P, R and F1 evaluate the pros and cons of the DTM algorithm can more accurately, comprehensively and intuitively. If the emitter emits a pulse train at a certain moment, but the DTM algorithm does not predict it correctly, the recall will decrease. If the algorithm predicts too many false pulse points, the precision will be reduced. Both statistics are indispensable.

C. Simulation Settings

The simulation includes two stages, training and testing. Before inputting the radar pulse data into the network, it is first segmented, and each segment is 60ms in length. In the training stage, the Encoder and Decoder convolutions are both with step size $step = 32$, and the kernel size is $k = 16$. RMM includes 2 deep separable dilated convolution blocks dual-path attention. Each convolution block contains $B' = 11$ dilated convolution structures. The dilation rate is increased from 2^0 to 2^{10}, and the size of the kernel is 3. The frame length L' of $\boldsymbol{Z} \in \mathcal{R}^{F' \times S \times L'}$ is 300. LayerNorm is used in the first block, and MeanNorm is used in the second one. The dimension of the pulse feature after Encoder is $F = 256$, which is compressed to $F' = 64$ after passing through the BottleNeck. BottleNeck consists of a $F \times F'$ linear layer and a $F' \times F'$ linear layer with PReLU activation. It maps the data dimension to a subspace, thus compressing the data.

The training process follows the paradigm of curriculum learning [41]. Curriculum learning assigns different weights to training samples of different difficulty according to the difficulty of the sample. In the initial stage, the weight of simple samples is the highest. As the training process continues, the weights of more difficult samples will be gradually increased. Such a process of dynamically assigning weights to samples is called curriculum. There are mostly simple samples at the initial stage of the curriculum, and the difficulty of the samples at the end of the curriculum increases. In the radar pulse deinterleaving task, the fixed PRI, noise-free two radar deinterleaving task is the simplest, and the unknown multiple PRI modulation modes, noise-containing radar pulse deinterleaving task of unknown number of radar emitters is the most difficult. The training includes three stages. First, we use the fixed and jitter PRI, 6 radar emitters, no missing data for pre-training. Then we use the 6 radar emitters data of multiple PRI modulation modes to train the RDN. The weight of jitter PRI is relative lower because it is relatively simple. At last, we use the configuration of random emitter number between 1 to 6, random selected PRI modulation modes and with noise $\theta_M = \theta_J = 0.3$ for training. In this way, the curriculum learning from easy to difficult is more conducive to the convergence of the network. And it reduces the radar pulse sequences length required for training to tens of minutes. The inclusion of a single radar pulse train ($K = 1$) in the training data allows the network to learn the characteristics of the deinterleaving radar pulse train itself more easily. It is helpful for sorting. In the first stage, learning rate $lr_1 = 0.001$. For the second and third stage, $lr_2 = lr_3 = 0.0001$. The number of training rounds *epoch* is 40 in each stages. Optimizers used in training are Adam. When F1 does not rise for two consecutive epochs, the learning rate is reduced to 0.7 times that of the previous epoch. The thresholds t_s and t_n belong to the hyper-parameters of the algorithm. We choose these two thresholds using grid search. The value is taken at an interval of 0.1 between 0 and 1. Then we take the thresholds with the highest F1 score on the validation dataset as the thresholds of the DTM. Finally we set the thresholds $t_s = 0.8$, $t_n = 0.5$.

In the simulation, we first evaluate the performance of the DTM algorithm in the jitter PRI dataset without any pre-processing and post-processing algorithms. We evaluate it in different settings including different radar emitters and different noise levels. Then we use some pre-processing and post-processing methods to observe its performance again. During pre-processing, it will assume that there is certain prior knowledge and use the generated data to fine tune the network. The fine-tuning method introduced above is also used. In the post-processing, the K-means and GMM are initialized by the prediction of the RDN. Then other PDW parameters $\boldsymbol{V}$ is used for re-clustering. Moreover, we compare the DTM algorithm with some commonly used deinterleaving algorithms to prove its accuracy and robustness. At last, we extend the DTM algorithm to deinterleave the radar pulse sequence of different PRI modulation modes under different conditions on the multiple PRI modulations dataset.

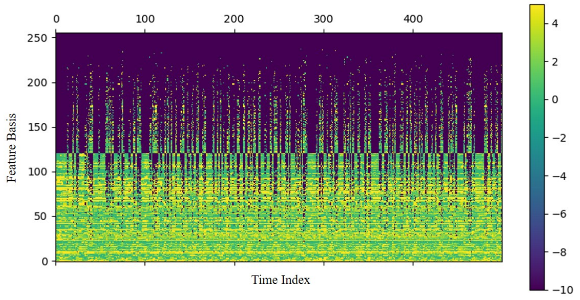


Fig. 7: Interleaving ideal ToA mask $\boldsymbol{M}$.

D. Results and Discussion

Without a special note, the algorithms are tested under the configuration of unknown number of emitters, unknown PRI and unknown PRI modulation mode.

The Encoder encodes an interleaved radar pulse frame to coefficient feature matrix $\boldsymbol{H}$ and then separates the components belonging to each emitter. The interleaving ideal ToA mask $\boldsymbol{M}$ in Fig. 7 visually indicates which radar emitter coefficient feature $\boldsymbol{H_k}$ dominates in the interleaving radar coefficient feature map $\boldsymbol{H}$. It is different from the ToA mask $\boldsymbol{M_k}$ used in the actual sorting. The interleaving ideal ToA mask points

This article has been accepted for publication in IEEE Transactions on Aerospace and Electronic Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TAES.2022.3193948

$M(i, j)$ takes discrete values from -10, 0, 1, 2, 3, 4 and 5. It indicate that different radar emitter k dominates the coefficient feature components (i, j) in $\boldsymbol{H}$. When the values of all $H_k(i, j)$ are small, $M(i, j) = -10$. When $H_k(i, j) > H_{-k}(i, j)$ and $H_k(i, j)$ is large enough, $M(i, j) = k$. $\boldsymbol{H_{-k}}$ represents the coefficient feature map of other radar emitters except $\boldsymbol{H_k}$. As mentioned earlier, $\boldsymbol{H} \in \mathcal{R}^{F \times L}$. Therefore, $\boldsymbol{M} \in \mathcal{R}^{F \times L}$. i is the subscript of the L dimension (also the "time" dimension) of $\boldsymbol{M}$, and j is the subscript of the F dimension (also the "frequency" dimension) of $\boldsymbol{M}$. It can be found that each coefficient feature point has different characteristics at different feature points, and these specific patterns are the basis for helping the network to identify each component. The coefficient mask points in the time dimension show a certain periodic pattern and are associated with certain emitters. Moreover, since radar emitters are arranged in ascending order by PRI, the radar coefficient mask points dominated by smaller ID k are also denser.

The appearance probability $P^{(s)}$ of each time point and the ground truth pulse point outputted by two radars after RDN processing are shown in Fig. 8 (a). This is in line with our previous assumption that the probability is the largest at the true pulse point, and part of the probability mass is allocated to the time point near the true pulse point. It is found from Fig. 8 (a) that even if the interval between two pulse trains of different emitters is very small, the algorithm can correctly sort them. We perform anomaly detection via removing the pulse points whose predicted PRI is beyond three times the standard deviation range by calculating the mean and variance of the PRI of the predicted deinterleaving sequence of each radar emitter. However, the deinterleaved sequence output by the RDN does not contain such abnormal pulse point. The deinterleaving results of the six radars are shown in Fig. 8 (b). The result is nearly the same as the ground truth, which proves that the DTM could accurately deinterleave pulse trains in a noisy environment with multiple emitters.

In an electronic warfare environment, there are often multiple radar emitters, and their number will change over time. This requires the algorithm to be able to dynamically predict the number of radars in a certain environment, and the prediction should be sufficiently accurate. Let the maximum number of radar emitters be $K_{max} = 6$. Fig. 9 shows the sorting results of the DTM algorithm under different numbers of emitters. When the number of radars continues to increase, the sorting effect will gradually deteriorate. But even when six radar emitters emit signals at the same time, the DTM algorithm has a deinterleaving accuracy of 90.6%. With such accuracy, the algorithm can easily estimate the actual PRI of each emitter and correct the predicted ToA train. Moreover, the RDN predicts the presence of each emitter with more than 99% confidence, and the number prediction accuracy reaches 100%. When the radar emitter k exists, $P_k^{(n)} > 0.99$. The number prediction accuracy is also calculated using the simulation results. The accuracy rate is 100%, that is, the RDN can always predict the number of radar emitters correctly. This shows that the algorithm does robustly respond to changes in the number of radars. It should be noted that the significance of accurately predicting the number of emitters is not only to improve the accuracy of the RDN deinterleaving, but more importantly, it also provides information for post-processing methods.

The curves in Fig. 10 are Precision-Recall (P-R) curves of sorting in different missing and jitter rates. The horizontal axis is the recall, and the vertical axis is the precision. As the threshold t_s increases from 0 to 1, precision gradually increases, and recall gradually decreases. Only a certain point in the P-R curve cannot fully measure the performance of our model. Through the overall performance of the P-R curve, the model could be evaluated more comprehensively. The larger the area under the P-R curve, the better the model performance. It can be seen from Fig. 10 that the presence of noise affects the performance of the model. Since the PRI is unknown, the model cannot determine which radar emitter is abnormal when the pulse points miss or deviate too much from the PRI mean μ_{pri}. Sometimes there is even a situation where multiple pulse points overlap each other, and it is more difficult to judge at this time. But even when $\theta_M = \theta_J = 0.3$, the accuracy of the RDN on the test set still reaches about 80%. Under such harsh conditions, assuming that loss and jitter are independent of each other, there is a 51% probability that the pulse point will be affected. For such pulse points, there is about 2/3 probability that they can be predicted correctly. The more specific statistics change with the missing and jitter rates are shown in Table III. When the missing and jitter rates are less than 0.1, noise has almost no effect on the performance of the RDN. Relatively speaking, the impact of PRI jitter on the DTM algorithm is greater than the missing. The above simulations prove that even without any pre-processing and post-processing, the DTM could robustly sort the unknown number of radar signals severely affected by noise in the electronic warfare environment.

Table IV compares the performance of the DTM algorithm with other commonly used radar pulse deinterleaving algorithms. It should be noted that because there is no universal dataset for radar pulse deinterleaving tasks, the data from the original paper may not be comparable. However, we guarantee that it is more difficult for deinterleaving in our simulation than the original one because of equal or stronger environment noise, unknown radar emitter number, and unknown specific jitter PRI information. In order to ensure the fairness of the comparison, the statistical data of the comparison algorithm in the table is the highest value of our implementation result and the result in the original paper [11, 18, 20, 23]. In Table IV, "fixed PRI" means that the average value of PRI μ_{pri} is fixed in the simulation. "2, w/o noise" means that the number of radar emitters is fixed at two, and there is no pulse missing and jitter. In the pre-processing process, we cache the processed radar pulse fragments, then use accurate labels (or post-processed labels with high confidence) to fine tune the network for 10 seconds, and then update the RDN parameters to perform prediction for the subsequent sequence of pulse trains. In the post-processing process, we use K-mean and GMM individually for re-clustering. The number of clusters is determined by the number predicted by RDN, and the initial centers of clusters are obtained by the weighted average of PDW parameter $\boldsymbol{V}$ with a confidence greater than 0.8. It should be noted that the PDW parameter $\boldsymbol{V}$ generated

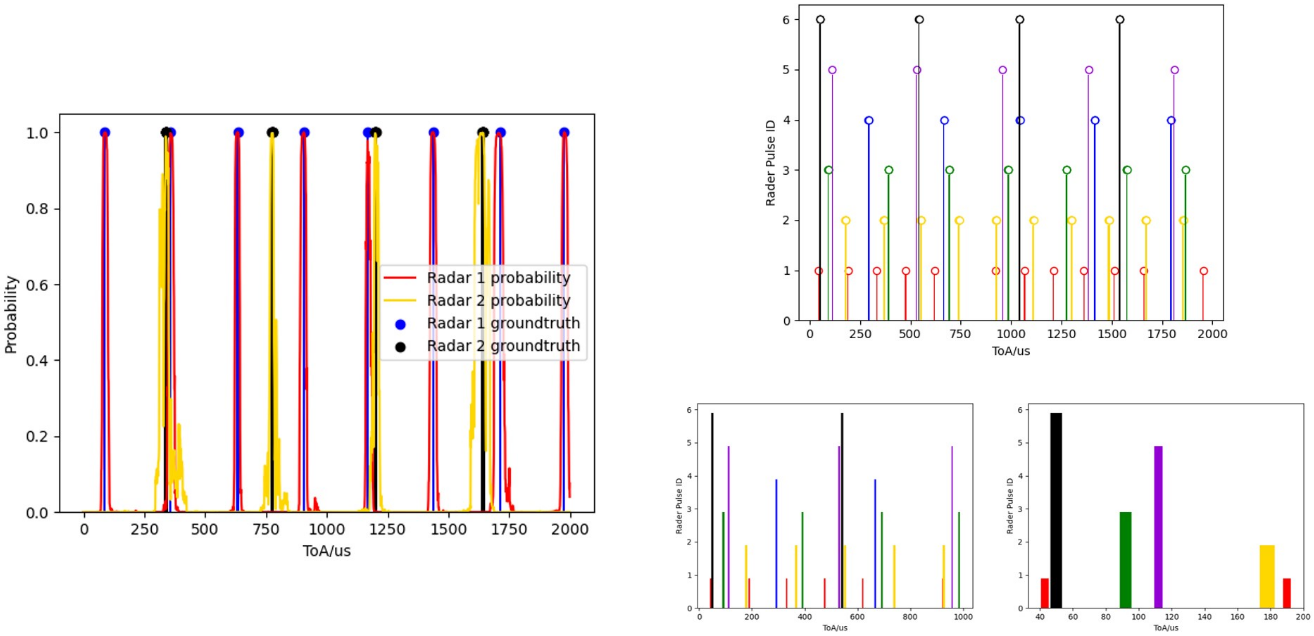


Fig. 8: (a) left: Probability $P_1^{(s)}$, $P_2^{(s)}$ and the real pulse points of them. (b) right: Sorting results (the two figures below show the details of the one above).

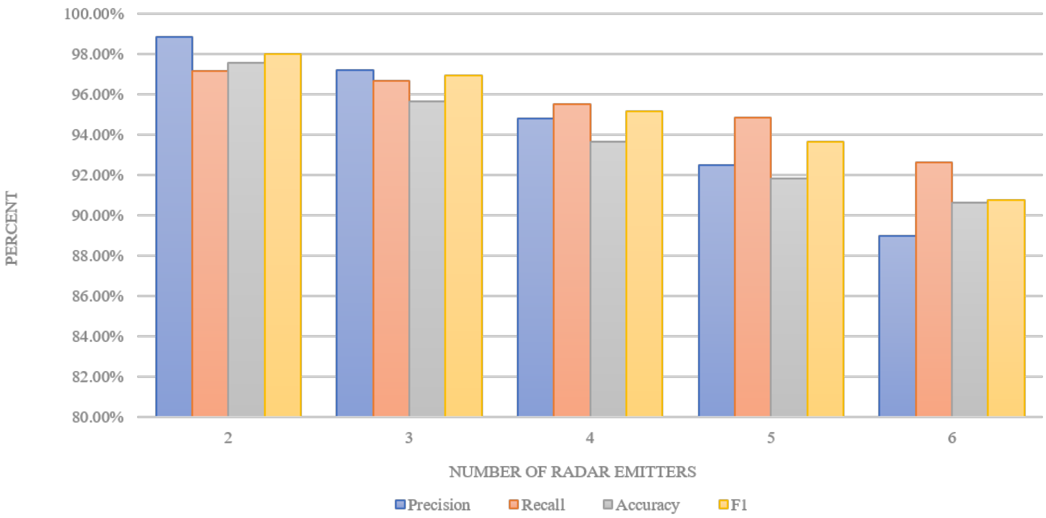


Fig. 9: The RDN deinterleaving statistics under different number of emitters.

by the simulation has a lot of noise and missing. Only when a sufficiently accurate result is obtained by ToA deinterleaving, the post-cluster would have a positive effect on sorting. The real-time rate of the whole DTM algorithm in test stage is less than 0.5, i.e., 60 ms radar pulse sequence can be processed within 30 ms. Therefore, the DTM algorithm can achieve real-time processin.

For clustering-based algorithms, K-means, GMM, and NN-cluster, they all use extra PDW feature $\boldsymbol{V}$, and the number of clusters is set to $K_{max} = 6$. These algorithms do not work well. The unknown and unfixed number of clusters influence their performance a lot. However, if the clustering algorithm is used as the post-processing module in the DTM algorithm, it could assist the RDN to make more accurate predictions because the number of clusters is known and there are initialized cluster centers. SDIF and IHM do not work well when the Signal-to-Noise Ratio (SNR) is low and the number of radars is large. With the increase of the number of radar emitters and the increase of environmental noise, the effect of the method based on sequence search will decrease sharply. Denoise Autoencoder (DA) can only deal with the fixed PRI situation and works well in it. The LSTM achieves good results when there are only two radar signals and they do not contain noise. In addition to using the ToA, the algorithm also uses Pulse Amplitude (PA) and some other features constructed through the ToA and PA as input. LSTM is so sensitive to the starting

This article has been accepted for publication in IEEE Transactions on Aerospace and Electronic Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TAES.2022.3193948

TABLE III: Statistical results of unknown number of radar varying noise rate

Noise Rate	Precision	Recall	Accuracy	F1
$\theta_M = 0.0, \theta_J = 0.0$	0.9205	0.9409	0.9496	0.9306
$\theta_M = 0.0, \theta_J = 0.1$	0.9104	0.9213	0.9323	0.9158
$\theta_M = 0.1, \theta_J = 0.0$	0.9129	0.9421	0.9468	0.9273
$\theta_M = 0.1, \theta_J = 0.1$	0.9063	0.9240	0.9303	0.9151
$\theta_M = 0.2, \theta_J = 0.2$	0.8312	0.8377	0.8617	0.8344
$\theta_M = 0.3, \theta_J = 0.3$	0.8146	0.7646	0.8172	0.7888

TABLE IV: Statistic results of deinterleaving with different algorithms

Method	Precision	Recall	Accuracy
K-Means [12]	/	/	0.3721
GMM [17]	/	/	0.7151
NN-cluster [18]	0.7021	0.7442	0.7573
SDIF [25, 42]	/	0.3587	/
IHM [11]	/	0.5120	/
DA [23] (fixed PRI)	0.8356	0.8452	0.8598
LSTM [20] (2, w/o noise)	0.9672	0.9836	0.9749
RDN (fixed PRI)	**0.9307**	**0.9642**	**0.9242**
RDN (2, w/o noise)	**0.9921**	**0.9791**	**0.9847**
RDN	**0.8146**	**0.7646**	**0.8172**
RDN + post. K-means	/	/	0.9441
RDN + post. GMM	/	/	0.9522
pre. + RDN	0.9166	0.8838	0.8907
pre. + RDN + post. GMM	/	/	**0.9764**

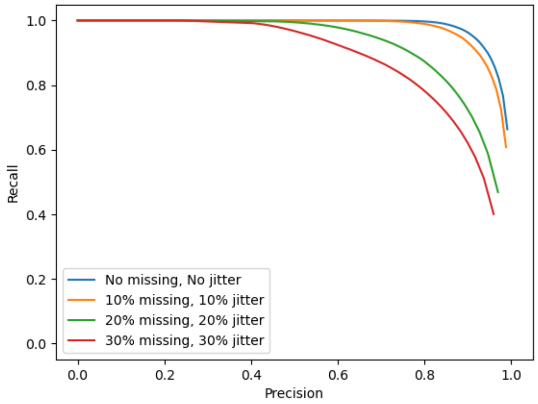


Fig. 10: Precision-Recall curves in different missing and jitter rates

point that it needs to be warmed up. After our investigation, none of the existing deep learning methods can achieve good results under the current settings. The RDN itself only uses the ToA train for pulse deinterleaving and achieves 81.72% accuracy. Simulations show that proper pre-processing and post-processing can further improve the prediction accuracy of the DTM algorithm to varying degrees. The DTM algorithm achieves the maximum accuracy of 97.64% among all the methods. The performance of the DTM is also better than the LSTM in the case of 2 emitters and no noise, and better than the DA in the fixed PRI condition. Then we assume that there is a prior knowledge about the modulation type and PRI respectively, and fine tune the network on this basis.

Finally, we performed a deinterleaving simulation in multiple PRI modulation modes modulated radar pulse sequence, as shown in Table V. We test the sorting accuracy of jitter PRI, sliding PRI, stagger PRI and group PRI under the configuration of unknown number of emitters, unknown PRI and unknown PRI modulation mode fisrt. It appears from the data generation process that all the generation parameters are randomly selected from the parameter range of Table II. The interleaving radar sequence contains multiple emitters, and the modulation type of each emitter is randomly selected from four modulation types. For an interleaving pulse sequence, it is possible that multiple radar emitters are of the same modulation type.

From the previous simulations of interleaved pulse sequences of jitter PRI, it can be found that when only the ToA information is used, robust and accurate sorting can be achieved for fixed PRI and jitter PRI pulse train, even when the number of radar emitters is large. From the current simulation, if the interleaved pulse sequence is generated by some emitters of multiple PRI modulation types, sorting with ToA alone cannot give very good results. The impact to accuracy of sliding PRI and jitter PRI is relatively small. Even using ToA alone can achieve a sorting accuracy of about 90%. This is because these two PRI modulation types basically belong to single-mode modulation and are less affected by ambiguity. Both stagger PRI and group PRI belong to multi-mode modulation, and the accuracy is low when only ToA is used for sorting. Therefore, when there is no additional information, the radar pulse sequences sorting accuracy with multi-mode modulation emitters is greatly reduce. The result of deinterleaving in this case is ambiguous. Despite the ambiguity, the DTM algorithm

TABLE V: Accuracy of the DTM algorithm in multiple PRI modulation modes with different information

PRI Modulation Types	No Extra Information	Modulation Types	PRI	Noisy PDW
Jitter PRI	0.8969	**0.9881**	0.9510	0.9298
Sliding PRI	0.9010	**0.9783**	0.9486	0.9262
Stagger PRI	0.6848	**0.9415**	0.9210	0.8079
Group PRI	0.6954	0.8241	**0.9485**	0.8530
All	0.8495	0.9444	**0.9491**	0.9083

can still sort the interleaved sequence by virtue of structures such as dilated convolution and dual-path attention. Since the missing rate and jitter rate are relatively small compared to the previous simulation ($\theta_M = \theta_J = 0.1$), the sorting accuracy of jitter PRI of the RDN has been improved. The sorting accuracy of sliding PRI is higher than the previous one, which is due to the easier predictability of its PRI change pattern. The deinterleaving accuracy of stagger PRI and group PRI is about 70%, which is much lower than jitter PRI and sliding PRI. It illustrates the negative impact of ambiguity on the deinterleaving of these two PRI modulation modes.

We considered three types of extra information for solving the ambiguity: modulation types, PRI and noisy PDW. ”Modulation Types” means that the modulation type of each radar emitter is known in advance, but the specific modulation parameters are not known. In this scenario, we have additional information about the emitter modulation type or we can infer it. That is to say, even if the modulation type is not known, it can be predicted accurately by deep learning algorithms [43]. ”PRI” refers to the fact that we know the PRI of each radar emitter in advance, which is a relatively strong additional information and is less likely to appear in practical situations. ”Noisy PDW” is the most readily available extra information, and is often used in radar pulse deinterleaving. Since extra PDW parameters other than ToA are usually noisy or even missing in our scenario, we focus on using ToA only for deinterleaving. Therefore, in the simulation, we regard the ”Noisy PDW” data as additional information, and use it to assist in solving the problem of radar signal deinterleaving under various pulse modulation types condition through the post-clustering method.

The PRI modulation information can increase the stagger PRI sorting accuracy of the DTM algorithm to more than 90%. There is also an increase of more than ten percentage points for group PRI. When we know the PRI information of the interleaved radar pulse sequence in advance, the DTM algorithm has high deinterleaving accuracy for any modulation mode, and the total accuracy reaches 94.91%. The only disadvantage is that when only the PRI information is known but the modulation type is not known, the fine-tuning of the network requires several minutes of samples. This also indirectly shows that the modulation type is very important information for the sorting of radar pulse sequences with multiple modulation modes. We can also treat each component A_k of stagger PRI and C_k of group PRI as a single one, then use noisy PDW information to assist deinterleaving. After sorting by the RDN, we re-cluster the sorted pulse sequences according to the correlation of each emitter or the correlation between PDW parameters to indirectly realize the sorting of stagger PRI and group PRI sequences. Since the RDN has a high sorting accuracy for sequences with single-mode PRI change patterns such as jitter PRI and sliding PRI, after converting stagger PRI and group PRI into multiple fixed PRIs, the sorting accuracy mainly depends on the noise level of other parameters of the emitters. Here we use K-means to re-integrate the sorted radar pulse sequence. Since other PDW parameters are noisy or missing, the sorting accuracy of stagger PRI and group PRI is lower than that sorting with modulation types or PRI information. But the advantage is that this method does not require any prior knowledge about the radar pulse emitters except for the PDW parameters.

When using the noisy PDW information, the constraint of the DTM algorithm on the multi-mode modulation radar emitter basically disappears. At this time, the sorting accuracy of stagger PRI and group PRI can reach more than 80%. If there is modulation type or PRI information, the deinterleaving results of stagger PRI and group PRI are not much different from single-mode modulation type. The modulation type information is relatively easy to obtain or infer from the ToA. At this point, the DTM algorithm can accurately deinterleave any PRI modulation type.

V. Conclusion

A DTM based sorting algorithm is proposed inspired by NMF in this paper. It is to solve the problem of radar deinterleaving in a complex electromagnetic environment with unknown PRI and the number of radar emitters only by the ToA feature. Even the PRI modulation mode is unknown, the DTM algorithm could deinterleave it. We also propose three methods to further improve the accuracy of the DTM algorithm in radar pulse sequence of multiple PRI modulation types.

The encoder-decoder structure and designed loss function of the RDN solves the feature extraction problem of non-stationary signals. The dilated convolution blocks of RMM increases the receptive field, and the deep separable convolution structure improves the sorting speed of the network. The dual-path attention mechanism integrates the PRI information within and between frames, which is more conducive to the identification and deinterleaving of different PRI modulations. The MeanNorm method makes full use of the deinterleaving ToA mask information outputted by the first convolution and attention block to reparameterize the pulse feature. The ToA mask based recursive method enables the RDN to obtain better results even when multiple pulse trains are missing and overlapped. Blind signal separation and NMF ideas enable the DTM to cope with interleaved radar signals of unknown PRI and unkown emitter number. The fine-tuning and the GMM based re-clustering method make the DTM be able to use the remaining parameters in the PDW and get more accurate deinterleaving results.

We prove the DTM algorithm can sort jitter PRI modulated radar pulse sequences of unknown emitter number and unknown PRI through simulations. It is robust to noise, with high deinterleaving accuracy even sorting only with ToA. It can run in real time and the RTF of the DTM is less than 0.5. Moreover, we also prove that the DTM algorithm could deinterleave multiple PRI modulation radar signals with a high accurate in the case of known PRI or unknown PRI but known PRI modulation types. Through the simulations, we can draw the conclusion that the DTM algorithm is better than those radar pulse deinterleaving benchmarks that are commonly used.

Whether pre-processing or post-processing, it can be used as a supplement and assistant to the RDN prediction, so that the whole DTM can get better results. However, it should be noted that the RDN prediction has performed very well without using any pre-processing and post-processing algorithms. The pre-processing and post-processing in the DTM cover a wide range of algorithms. We only give a few examples here. In future work, we will continue to explore the network structure of the DTM based on ToA sorting and more other related post-processing methods. On the other hand, we will continue to explore how to decouple the PRI modulation type identification and radar pulse deinterleaving, so as to predict the PRI modulation type of each radar emitter first, and then deinterleave the pulse sequence according to it.

Acknowledgment

This work was supported in part by the National Key RD Program of China under Grant 2021ZD0201300, and by the National Science Foundation of China under Grant 61876076.

References

[1] S. Q. Wang, G. P. Hu, Q. L. Zhang, C. Y. Gao, and T. Cai, "The background and significance of radar signal sorting research in modern warfare," *Procedia Computer Science*, vol. 154, pp. 519–523, 2019.

[2] S. Q. Wang, J. Bai, X. Y. Huang, C. Y. Gao, and P. F. Wan, "Analysis of radar emitter signal sorting and recognition model structure," *Procedia Computer Science*, vol. 154, pp. 500–503, 2019.

[3] R. Wiley, *ELINT: The interception and analysis of radar signals*. Artech, 2006.

[4] H. Hasani and M. R. Khosravi, "Pulse deinterleaving based on fusing pdws and pri extraction process for radar-assisted edge devices considering computational costs," *EURASIP Journal on Wireless Communications and Networking*, vol. 2021, no. 1, pp. 1–14, 2021.

[5] H. Mardia, "New techniques for the deinterleaving of repetitive sequences," in *IEE Proceedings F (Radar and Signal Processing)*, vol. 136, no. 4. IET, 1989, pp. 149–154.

[6] H. Li, Y. Han, Y. Cai, and R.-h. Tao, "Overview of the crucial technology research for radar signal sorting," *Systems Engineering and Electronics*, vol. 27, no. 12, pp. 2035–2040, 2005.

[7] Y. Liu and Q. Zhang, "An improved algorithm for pri modulation recognition," in *2017 IEEE International Conference on Signal Processing, Communications and Computing (ICSPCC)*. IEEE, 2017, pp. 1–5.

[8] A. Mahdavi and A. M. Pezeshk, "A robust method for pri modulation recognition," in *IEEE 10th international conference on signal processing proceedings*. IEEE, 2010, pp. 1873–1876.

[9] S. Sridharan, P. Nnssrk, G. Riya, and M. Brindha, "Improved pulse repetition interval (pri) deinterleaving for electronic support measure (esm) receiver," *Int. Journal of Advanced Computing and Electronics Technology (IJACET)*, vol. 2, no. 3, pp. 37–43, 2015.

[10] M. Bagheri and M. H. Sedaaghi, "A new method for detecting jittered pri in histogram-based methods," *Turkish Journal of Electrical Engineering & Computer Sciences*, vol. 26, no. 3, pp. 1214–1224, 2018.

[11] K. Manickchand, J. J. Strydom, and A. K. Mishra, "Comparative study of toa based emitter deinterleaving and tracking algorithms," in *2017 IEEE AFRICON*. IEEE, 2017, pp. 221–226.

[12] X. Feng, X. Hu, and Y. Liu, "Radar signal sorting algorithm of k-means clustering based on data field," in *2017 3rd IEEE International Conference on Computer and Communications (ICCC)*. IEEE, 2017, pp. 2262–2266.

[13] K. Gençol, A. Kara, and N. At, "Improvements on deinterleaving of radar pulses in dynamically varying signal environments," *Digital Signal Processing*, vol. 69, pp. 86–93, 2017.

[14] S. Wang, C. Gao, Q. Zhang, V. Dakulagi, H. Zeng, G. Zheng, J. Bai, Y. Song, J. Cai, and B. Zong, "Research and experiment of radar signal support vector clustering sorting based on feature extraction and feature selection," *IEEE Access*, vol. 8, pp. 93 322–93 334, 2020.

[15] M. Mottier, G. Chardon, and F. Pascal, "Deinterleaving and clustering unknown radar pulses," in *2021 IEEE Radar Conference (RadarConf21)*. IEEE, 2021, pp. 1–6.

[16] J. Young, A. Høst-Madsen, and E.-M. Nosal, "Deinterleaving of mixtures of renewal processes," *IEEE Transactions on Signal Processing*, vol. 67, no. 4, pp. 885–898, 2018.

[17] X. Gong, H. Meng, and X. Wang, "A gmm-based algorithm for classification of radar emitters," in *2008 9th International Conference on Signal Processing*. IEEE, 2008, pp. 2434–2437.

[18] S. Gasperini, M. Paschali, C. Hopke, D. Wittmann, and N. Navab, "Signal clustering with class-independent segmentation," in *ICASSP 2020-2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2020, pp. 3982–3986.

[19] Q. Guo, L. Teng, L. Qi, X. Ji, and J. Xiang, "A novel radar signals sorting method-based trajectory features," *IEEE Access*, vol. 7, pp. 171 235–171 245, 2019.

This article has been accepted for publication in IEEE Transactions on Aerospace and Electronic Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TAES.2022.3193948

[20] Z.-M. Liu and S. Y. Philip, "Classification, denoising, and deinterleaving of pulse streams with recurrent neural networks," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 55, no. 4, pp. 1624–1639, 2018.

[21] J. Mun, S. Ha, and J. Lee, "Automotive radar signal interference mitigation using rnn with self attention," in *ICASSP 2020-2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, 2020, pp. 3802–3806.

[22] F. Li, Z. Yang, and C. Yang, "Radar signal sorting technology based on image processing and hough transform," in *2018 International Conference on Microwave and Millimeter Wave Technology (ICMMT)*. IEEE, 2018, pp. 1–3.

[23] X. Li, Z. Liu, and Z. Huang, "Deinterleaving of pulse streams with denoising autoencoders," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 56, no. 6, pp. 4767–4778, 2020.

[24] C. Ren, J. Cao, Y. Fu, and K. E. Barner, "Improved method for pulse sorting based on pri transform," in *Signal Processing, Sensor/Information Fusion, and Target Recognition XXIII*, vol. 9091. International Society for Optics and Photonics, 2014, p. 90911T.

[25] Z. Ge, X. Sun, W. Ren, W. Chen, and G. Xu, "Improved algorithm of radar pulse repetition interval deinterleaving based on pulse correlation," *IEEE Access*, vol. 7, pp. 30 126–30 134, 2019.

[26] Y. Mao, J. Han, G. Guo, and X. Qing, "An improved algorithm of pri transform," in *2009 WRI Global Congress on Intelligent Systems*, vol. 3. IEEE, 2009, pp. 145–149.

[27] J. A. Anderson, M. T. Gately, P. A. Penz, and D. R. Collins, "Radar signal categorization using a neural network," *Proceedings of the IEEE*, vol. 78, no. 10, pp. 1646–1657, 1990.

[28] J.-W. Han and C. H. Park, "A unified method for deinterleaving and pri modulation recognition of radar pulses based on deep neural networks," *IEEE Access*, vol. 9, pp. 89 360–89 375, 2021.

[29] G. Jajoo, Y. Kumar, S. K. Yadav, B. Adhikari, and A. Kumar, "Blind signal modulation recognition through clustering analysis of constellation signature," *Expert Systems with Applications*, vol. 90, pp. 13–22, 2017.

[30] X. Wang, G. Huang, Z. Zhou, W. Tian, J. Yao, and J. Gao, "Radar emitter intrapulse signal blind sorting under modified wavelet denoising," *The Journal of Engineering*, vol. 2019, no. 21, pp. 8013–8017, 2019.

[31] S. Ikeda and N. Murata, "A method of ica in time-frequency domain," in *in Proc. ICA*, 1999.

[32] D. D. Lee and H. S. Seung, "Learning the parts of objects by non-negative matrix factorization," *Nature*, vol. 401, no. 6755, pp. 788–791, 1999.

[33] Y. Luo and N. Mesgarani, "Conv-tasnet: Surpassing ideal time–frequency magnitude masking for speech separation," *IEEE/ACM transactions on audio, speech, and language processing*, vol. 27, no. 8, pp. 1256–1266, 2019.

[34] J. Wen, F. Xiongjun, C. Jiayun, and Q. Rui, "An improved de-interleaving algorithm of radar pulses based on sofm with self-adaptive network topology," *Journal of Systems Engineering and Electronics*, vol. 31, no. 4, pp. 712–721, 2020.

[35] M. Sundermeyer, R. Schlüter, and H. Ney, "Lstm neural networks for language modeling," in *Thirteenth annual conference of the international speech communication association*, 2012.

[36] E. Brookner, "Phased arrays and radars-past, present and future," *Microwave Journal, Cover Feature*, 2006.

[37] Z. Yang, Z. Yuan, and J. Laaksonen, "Projective non-negative matrix factorization with applications to facial image processing," *International Journal of Pattern Recognition and Artificial Intelligence*, vol. 21, no. 08, pp. 1353–1362, 2007.

[38] J. Kim and H. Park, "Sparse nonnegative matrix factorization for clustering," Georgia Institute of Technology, Tech. Rep., 2008.

[39] F. Yu and V. Koltun, "Multi-scale context aggregation by dilated convolutions," 2016.

[40] J. L. Ba, J. R. Kiros, and G. E. Hinton, "Layer normalization," *arXiv preprint arXiv:1607.06450*, 2016.

[41] Y. Bengio, J. Louradour, R. Collobert, and J. Weston, "Curriculum learning," in *Proceedings of the 26th annual international conference on machine learning*, 2009, pp. 41–48.

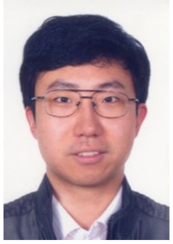
[42] Y. Xi, Y. Wu, X. Wu, and K. Jiang, "An improved sdif algorithm for anti-radiation radar using dynamic sequence search," in *2017 36th Chinese Control Conference (CCC)*. IEEE, 2017, pp. 5596–5601.

[43] S. Wei, Q. Qu, Y. Wu, M. Wang, and J. Shi, "Pri modulation recognition based on squeeze-and-excitation networks," *IEEE Communications Letters*, vol. 24, no. 5, pp. 1047–1051, 2020.

This article has been accepted for publication in IEEE Transactions on Aerospace and Electronic Systems. This is the author's version which has not been fully edited and content may change prior to final publication. Citation information: DOI 10.1109/TAES.2022.3193948

Haoran Xiang Haoran Xiang received the B.Sc. degrees in spatial information and digital technology from University of Electronic Science and Technology, Chendu, China, in 2020. He is currently pursuing the M.Sc. degree in computer science and technology from Nanjing University, Nanjing, China. His current research interests include signal separation and enhancement, speech recognition, and speech synthesis.

Furao Shen Furao Shen received the B.Sc. and M.Sc. degrees in mathematics from Nanjing University, Nanjing, China, in 1995 and 1998, respectively, and the Ph.D. degree from the Tokyo Institute of Technology, Tokyo, Japan, in 2006. He is currently a Full Professor of computer science and technology with Nanjing University. His current research interests include neural computing and robotic intelligence.



Jian Zhao Jian Zhao received the B.S. degree from Nanjing University, Nanjing, China, in 2001, the M.Sc. degree from Hamburg University of Technology, Hamburg, Germany, in 2004, and the Ph.D. degree in electrical engineering from Swiss Federal Institute of Technology (ETH) Zurich, Switzerland, in 2010. Since December 2010, he has been with the Institute for Infocomm Research, Singapore. His research interests include optimization techniques in wireless communications, multiuser MIMO communications, and cooperative communications. Dr. Zhao has received a number of awards, including the DAAD-Siemens Asia 21st Century Scholarship, IEEE Globecom 2008 Best Paper Award, and Chinese Government Award for Outstanding Self-Financed Students Abroad.