Throughput Maximization With Rate-Dependent Power Consumption in Battery-Limited Multiuser Networks

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***Abstract*—This article proposes beamforming strategies for multiuser communication networks by jointly considering the data rate and battery lifetime of each user. We consider a practical power consumption model, where the decoding power of each user device is a nonlinear function of the data rate. We assume each user device is powered by a battery with limited capacity. The power consumption of the user device determines the expected battery lifetime which conforms to Peukert's law. Two non-convex optimization problems are considered based on two criteria: maximizing the minimum of user throughputs (MaxMin) and maximizing the weighted sum of user throughputs (MaxSum), during the battery lifetime. For the MaxMin problem, we solve it by using a bisection search algorithm. For the MaxSum problem, we propose an algorithm based on the successive convex approximation, which can find a stable point that approximates the optimum solution. Numerical simulations verify that the expected data throughput of the multiuser system can be greatly improved by properly choosing the beamforming strategies using the proposed algorithms.**

***Index Terms*—Throughput maximization, multiuser communications, rate-dependent power consumption, beamforming, battery lifetime.**

I. Introduction

In modern wireless networks, the mobile devices are powered by batteries with limited capacities. The power consumption of each mobile device also depends on the decoding data rate. Therefore, increasing the transmission data rate will not necessarily increase the received data throughput because it also reduces the battery lifetime. In order to maximize the expected data throughput of battery-limited devices, we must jointly consider the battery lifetime and the data rate.

The authors of [1] considered maximizing the total battery lifetime under rate constraints in a device-to-device (D2D) network. They formulated the problem as a resource allocation game and proposed a resource auction algorithm. A similar problem was considered in [2], where an uplink resource sharing scenario was considered in a D2D communication network. To maximize the sum data throughput during the lifetime of all users, the authors formulated such a problem as a power control game and introduced an iterative combinatorial auction algorithm. The studies in [1], [2] considered single-antenna transmitters and receivers. The authors of [3] solved a problem of maximizing the total throughput and device lifetime for a point-to-point multiple-input multiple-output (MIMO) system, through the asymptotic analysis.

In this paper, we consider a downlink multiuser communication network, where a multi-antenna transmitter sends independent data streams to single-antenna devices (or users), simultaneously. Each device is powered by a battery with limited capacity. The power consumption for the decoding at the battery-limited device is a nonlinear convex function of the data rate [4], [5]. The data rate of each device is determined by the beamforming strategy at the transmitter. By properly choosing the beamforming strategies, we can adjust the expected battery lifetime and hence the expected data throughput of each device. We consider two performance criteria for the system, i.e., maximizing the minimum of the expected data throughput of all devices (called *MaxMin*) and maximizing the sum of the expected data throughput for all devices (called *MaxSum*). Efficient algorithms are proposed to solve the two network optimization problems. Our contributions can be summarized as follows:

- We consider a novel *rate-dependent power consumption model* and take into account its impact on the *nonlinear battery lifetime* for optimizing the data throughput. To the best of our knowledge, jointly considering the rate-dependent power consumption and the nonlinear battery lifetime model is a novel approach that cannot be found in the existing literature.
- For the *multi-antenna multiuser networks*, we consider the throughput maximization problems, in which the beamforming strategies are optimized. Solving the optimization problems is nontrivial, compared to the existing problems for the single-antenna networks.
- We consider two important optimization criteria, namely MaxMin and MaxSum, for throughput maximization in the multi-antenna multiuser networks. To tackle the nontrivial MaxMin and MaxSum problems, we propose efficient and effective algorithms based on the bisection search and the successive convex approximation (SCA).

Notation: For a matrix or a vector, $(\cdot)^*$ and $(\cdot)^H$ stand for the conjugate and the conjugate transpose, respectively. $\mathbb{C}^{a\times b}$ denotes the complex vector or matrix with the dimension of a-by-b. $|\cdot|$ and $\|\cdot\|$ stand for the absolute value of a complex number and the Euclidean norm of a vector, respectively. $x^{(n)}$ denotes the value of x in the nth iteration.

II. System Model

In the considered multi-antenna multiuser networks, a transmitter with M antennas sends multiple data streams to N single-antenna users simultaneously in the same frequency channel. We consider a low-mobility environment. The channel from the transmitter to the ith user and its beamforming vector are denoted by $\mathbf{h}_i^H \in \mathbb{C}^{1\times M}, i = 1,\ldots,N$, and $\mathbf{v}_i \in \mathbb{C}^{M\times 1}$, respectively. Here, the transmit power is bounded by P. The received signal-to-interference-plus-noise ratio (SINR) at the

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ith user can then be expressed as

$$\mathsf{SINR}_i(\{\mathbf{v}_i\}) = \frac{\left|\mathbf{h}_i^H \mathbf{v}_i\right|^2}{\sum_{j\neq i}\left|\mathbf{h}_i^H \mathbf{v}_j\right|^2 + \sigma^2} \tag{1}$$

where σ^2 is the noise variance at each user. The corresponding achievable rate, which is the upper bound of the data rate, can be written as

$$R_i = \log_2\left(1 + \mathsf{SINR}_i(\{\mathbf{v}_i\})\right). \tag{2}$$

The power consumption at the user side consists of two parts, the *circuit power* and the *decoding power*. The circuit power is mainly consumed by all the circuit blocks. Without loss of generality, it is assumed to be static as p_i for user i. On the other hand, the decoding power of the ith user is consumed for decoding the data bits received from the transmitter. The decoding power can be modeled as a function of its *decoding data rate* r_i as br_i^m [4], [5], where b and m $(m \geq 1)$ are the rate-dependent power consumption coefficients. Therefore, the total power consumption of the ith user can be modeled as follows:

$$P_i = p_i + br_i^m. \tag{3}$$

As mentioned, mobile users are usually powered by batteries with limited capacities, which is a critical bottleneck for wireless communications. To capture this issue, the nonlinear effects must be considered for battery discharges. A widely used empirical model for battery lifetime estimation is Peukert's law [2], [6]. According to Peukert's law, the expected battery lifetime of the ith user can be calculated as

$$T_i = \frac{C_i}{I_i^a} = \frac{C_i}{(P_i/U_i)^a} = \frac{C_i U_i^a}{(p_i + br_i^m)^a} \tag{4}$$

where C_i and U_i denote the battery capacity and the operating voltage of user i, respectively, and a $(a > 1)$ is the battery energy decay exponent. Peukert's law implies that there exists a fundamental tradeoff between the battery life and the decoding data rate. Considering Peukert's law, the expected *decoding data throughput* of the ith user, which is the expected amount of data bits received during its battery life, can be regarded as a function of r_i as follows:

$$d_i(r_i) = r_i T_i = \frac{r_i}{(p_i + br_i^m)^a} C_i U_i^a. \tag{5}$$

Such a metric $d_i(r_i)$ takes into account the tradeoff between the battery life and the decoding data rate, and it is a more practical performance criterion than the achievable rate R_i for battery-limited wireless communications.

III. Optimization Problems

Two optimization problems, namely maximizing the minimum of the expected data throughput of all users (MaxMin) and maximizing the sum of the expected data throughput for all users (MaxSum) during the battery lifetime, are formulated and solved in the sequel.

A. MaxMin Problem

To guarantee fairness among all users, the MaxMin problem is considered. The MaxMin optimization is an egalitarian approach, which guarantees as much as possible data throughput to each user of the network. This is relevant, e.g., in networks where the data volumes that need to be delivered to each user are similar. We design the beamforming vector $\mathbf{v}_i$ and decoding data rate r_i for user i, $\forall i = 1,\ldots,N$, based on the MaxMin criterion. Such an optimization problem can be formulated as

$$\max_{\{r_i\},\{\mathbf{v}_i\}} \min_{1\leq i\leq N} d_i(r_i) \tag{6a}$$

$$\text{s.t.} \quad r_i \leq R_i, \quad \forall i = 1,\ldots,N \tag{6b}$$

$$\sum_{i=1}^{N} \|\mathbf{v}_i\|^2 \leq P \tag{6c}$$

where the objective function in (6a) is the minimum data throughput during the battery life; the first constraint (6b) comes from the fact that the decoding rate is no greater than the achievable rate, and the second constraint (6c) is for the transmit power limit. Using (2) and an auxiliary variable t, the non-convex optimization problem (6) can be equivalently converted to

$$\max_{t,\{r_i\},\{\mathbf{v}_i\}} \quad t \tag{7a}$$

$$\text{s.t.} \quad d_i(r_i) \geq t, \quad \forall i = 1,\ldots,N \tag{7b}$$

$$2^{r_i} \leq 1 + \mathsf{SINR}_i(\{\mathbf{v}_i\}), \quad \forall i \tag{7c}$$

$$\text{(6c)}. \tag{7d}$$

The auxiliary variable t serves as a lower-bound threshold for the data throughput of all users. By maximizing t, we equivalently maximize the minimum throughput of all users.

The throughput $d_i(r_i)$ is a nonlinear non-convex function of r_i. Thus (7) is still difficult to solve. The following property will be used to devise an efficient algorithm to tackle the MaxMin problem in (7):

Property 1: The decoding data throughput $d_i(r_i)$ is a unimodal function of r_i, which has only one peak value. The critical point r_i^{sta} and the corresponding maxima $d_i(r_i^{\text{sta}})$ can be respectively expressed as

$$r_i^{\text{sta}} = \sqrt[m]{\frac{p_i}{(am-1)b}}, \tag{8a}$$

$$d_i(r_i^{\text{sta}}) = \frac{(am-1)^{(a-\frac{1}{m})}}{p_i^{(a-\frac{1}{m})}(am)^a b^{\frac{1}{m}}} \cdot C_i U_i^a. \tag{8b}$$

Proof: The first derivative of $d_i(r_i)$ with respect to r_i is as follows:

$$d_i'(r_i) = \frac{\partial d_i(r_i)}{\partial r_i} = \frac{p_i - b(am-1)r_i^m}{(p_i + br_i^m)^{(a+1)}} C_i U_i^a. \tag{9}$$

The function $d_i(r_i)$ has a single critical point and is maximized when $d_i'(r_i) = 0$. The critical point r_i^{sta} and the corresponding maxima $d_i(r_i^{\text{sta}})$ can be derived as (8a) and (8b), respectively. Moreover, from the fact that $d_i'(r_i) > 0$ if $r_i < r_i^{\text{sta}}$ and $d_i'(r_i) < 0$ otherwise, we can conclude that $d_i(r_i)$ is a unimodal function of r_i. This is numerically verified in Fig. 1 as well. ■

Based on Property 1, the optimum solution of t in (7) can only lie in the range $[0, \min_i d_i(r_i^{\text{sta}})]$. Moreover, for each user i, we only need to consider the data rate range of $r_i \in [0, r_i^{\text{sta}}]$ for the achievable data throughput because the throughput is monotonically decreasing outside that range. We propose a bisection method to obtain the optimum solution of (7) as in Algorithm 1. Note that the proposed method can be readily extended to incorporate the quality-of-service (QoS) constraints into the MaxMin problem.

Algorithm 1 first initializes the search space of t, i.e., $[t_{\min}, t_{\max}]$, to the possible range of the optimum solution for t in Step 1. In each iteration, we set the value t as the target data throughput. In Step 4, we find

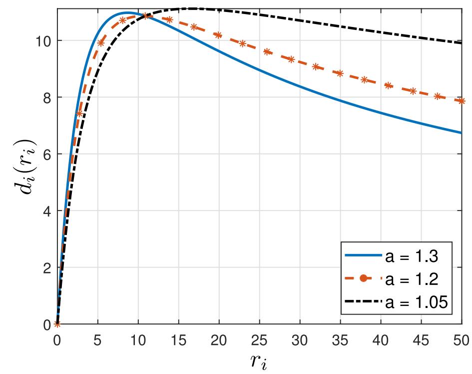

Fig. 1. Decoding data throughput function $d_i(r_i)$ when $p_i = 0.3$, $b = 0.04$, $m = 1.2$. Here $C_i U_i^a$ are normalized to 1.

Algorithm 1: Bisection Method for the MaxMin Problem.

1: Initialize $t_{\min} = 0$ and $t_{\max} = \min_i d_i(r_i^{\text{sta}})$;
2: **while** $t_{\max} - t_{\min} > \epsilon$ **do**
3: Set $t = (t_{\min} + t_{\max})/2$;
4: Find the numerical solution $\{r_i\}$ for the nonlinear equation $d_i(r_i) = t$, where $r_i \in [0, r_i^{\text{sta}}]$, using the Newton-Raphson method [7, Section 7.1];
5: Solve the following optimization problem for the obtained solution $\{r_i\}$:

$$\min_{\{\mathbf{v}_i\}} \sum_{i=1}^{N} \|\mathbf{v}_i\|^2$$
$$\text{s.t. } \mathsf{SINR}_i(\{\mathbf{v}_i\}) \geq 2^{r_i} - 1, \quad \forall i. \tag{10}$$

6: **if** the solution of (10) satisfies $\sum_i \|\mathbf{v}_i\|^2 \leq P$, i.e., the data rates $\{r_i\}$ are jointly achievable by all the users **then**
7: Set $t_{\min} = t$; store the feasible $\{\mathbf{v}_i\}$;
8: **else**
9: Set $t_{\max} = t$;
10: **end if**
11: **end while**

the corresponding data rate r_i for the data throughput t by solving the nonlinear equation in the range of $r_i \in [0, r_i^{\text{sta}}]$. According to Property 1, there exists only one solution for each equation in the considered range of r_i. In Step 5 and 6, we test whether such data rates $\{r_i\}$ obtained in Step 4 are jointly achievable for all users by checking the minimum required transmit power in (10). Such a problem can be converted to a second-order cone programming (SOCP) problem [8] and readily solved by fast iterative algorithms [9]. By iteratively updating the value of t using the bisection method, we obtain the maximum value of t that is jointly achievable for all users. Furthermore, we introduce the following remark:

Remark 1: When ϵ is sufficiently small, the result produced by Algorithm 1 is sufficiently close to a global optimum solution of (6).

Proof: See Appendix A. ■

B. MaxSum Problem

Focusing on the whole network performance rather than the fairness, the MaxSum approach aims to maximize the sum throughput of all users. The MaxSum problem is formulated as:

$$\max_{\{r_i\},\{\mathbf{v}_i\}} \sum_{i=1}^{N} w_i d_i(r_i) \tag{11a}$$
$$\text{s.t. } \text{(6b) and (6c)} \tag{11b}$$

where $w_i > 0$ denotes the weight for the ith user data throughput. Since the objective function of (11) is neither convex nor concave, it is also intractable to directly solve the MaxSum problem. By introducing additional variables $\{t_i\}, \{\Gamma_i\}, \{z_i\}$ and denoting the set of optimization variables as $\mathbf{\Omega} = \{\{r_i\}, \{\mathbf{v}_i\}, \{t_i\}, \{\Gamma_i\}, \{z_i\}\}$, we can equivalently reformulate (11) as:

$$\max_{\mathbf{\Omega}} \sum_{i=1}^{N} w_i t_i C_i U_i^a \tag{12a}$$
$$\text{s.t. } t_i \leq \frac{r_i}{z_i}, \quad \forall i = 1, \ldots, N \tag{12b}$$
$$r_i \leq \log_2(1 + \Gamma_i), \quad \forall i \tag{12c}$$
$$z_i \geq (p_i + b r_i^m)^a, \quad \forall i \tag{12d}$$
$$\sum_{j \neq i} \left|\mathbf{h}_i^H \mathbf{v}_j\right|^2 + \sigma^2 \leq \frac{\left|\mathbf{h}_i^H \mathbf{v}_i\right|^2}{\Gamma_i}, \quad \forall i \tag{12e}$$
$$\text{(6c).} \tag{12f}$$

Moreover, the constraint (12b) can be equivalently written as

$$\frac{(t_i + z_i)^2}{4} \leq \frac{(t_i - z_i)^2}{4} + r_i, \quad \forall i. \tag{13}$$

Because the right-hand-side (RHS) of (13) and (12e) are both convex functions, the problem (12) is still non-convex. We propose to replace the RHS functions of (13) and (12e) with their first-order approximations as the surrogate functions. In particular, due to the convexity of both functions, we have

$$\varphi^{(n)}(t_i, z_i, r_i) \triangleq \frac{t_i^{(n)} - z_i^{(n)}}{2}(t_i - z_i) + r_i - \frac{\left(t_i^{(n)} - z_i^{(n)}\right)^2}{4}$$
$$\leq \text{RHS of (13)}, \quad \forall i \tag{14}$$

$$\zeta^{(n)}(\mathbf{v}_i, \Gamma_i) \triangleq \frac{2\,\text{Re}\left\{\left(\mathbf{h}_i^H \mathbf{v}_i^{(n)}\right)^* \mathbf{h}_i^H \mathbf{v}_i\right\}}{\Gamma_i^{(n)}} - \frac{\left|\mathbf{h}_i^H \mathbf{v}_i^{(n)}\right|^2}{\left(\Gamma_i^{(n)}\right)^2}\Gamma_i$$
$$\leq \text{RHS of (12e)}, \quad \forall i \tag{15}$$

where $\{t_i^{(n)}\}$, $\{z_i^{(n)}\}$, $\{\Gamma_i^{(n)}\}$, and $\{\mathbf{v}_i^{(n)}\}$ are the given values in the nth iteration (the iterative algorithm will be shown shortly). The functions $\varphi^{(n)}(t_i, z_i, r_i)$ and $\zeta^{(n)}(\mathbf{v}_i, \Gamma_i)$ are the first-order approximations of the RHS functions of (13) and (12e) around $(t_i^{(n)}, z_i^{(n)})$ and $(\mathbf{v}_i^{(n)}, \Gamma_i^{(n)})$, respectively.

By substituting $\varphi^{(n)}(t_i, z_i, r_i)$ and $\zeta^{(n)}(\mathbf{v}_i, \Gamma_i)$ for the RHS functions of (13) and (12e), the problem (12) can be approximated by the following problem in the $(n+1)$th iteration:

$$\max_{\mathbf{\Omega}} \sum_{i=1}^{N} w_i t_i C_i U_i^a \tag{16a}$$
$$\text{s.t. } \frac{(t_i + z_i)^2}{4} \leq \varphi^{(n)}(t_i, z_i, r_i), \quad \forall i = 1, \ldots, N \tag{16b}$$
$$\sum_{j \neq i} \left|\mathbf{h}_i^H \mathbf{v}_j\right|^2 + \sigma^2 \leq \zeta^{(n)}(\mathbf{v}_i, \Gamma_i), \quad \forall i \tag{16c}$$
$$\text{(12c), (12d), and (6c).} \tag{16d}$$

Algorithm 2: SCA Method for the MaxSum Problem.

1: Choose random initialization of the beamforming vectors $\{\mathbf{v}_i^{(0)}\}$ such that they can fulfill the power constraints: $\sum_{i=1}^{N} \|\mathbf{v}_i^{(0)}\|^2 \le P$;
2: Initialize $\Gamma_i^{(0)}$ as $\Gamma_i^{(0)} = \frac{|\mathbf{h}_i^H \mathbf{v}_i^{(0)}|^2}{\sum_{j\neq i} |\mathbf{h}_i^H \mathbf{v}_j^{(0)}|^2+\sigma^2}$, $\forall i$;
3: Calculate $r_i^{(0)} = \log_2(1+\Gamma_i^{(0)})$, $\forall i$;
4: Initialize $z_i^{(0)}$ as $z_i^{(0)} = (p_i + b(r_i^{(0)})^m)^a$, $\forall i$;
5: Initialize $t_i^{(0)}$ as $t_i^{(0)} = r_i^{(0)}/z_i^{(0)}$, $\forall i$;
6: Set $n=0$; $\text{obj}^{(0)}=0$;
7: **repeat**
8: For the given values of $\{t_i^{(n)}\}$, $\{z_i^{(n)}\}$, $\{\Gamma_i^{(n)}\}$, and $\{\mathbf{v}_i^{(n)}\}$, solve (16) to obtain the solutions for $\{\{r_i\}, \{\mathbf{v}_i\}, \{t_i\}, \{\Gamma_i\}, \{z_i\}\}$;
9: Compute $\text{obj}^{(n+1)} = \sum_{i=1}^{N} w_i t_i C_i U_i^a$ and $\Delta = |\text{obj}^{(n+1)} - \text{obj}^{(n)}|$;
10: Let $\mathbf{v}_i^{(n+1)} = \mathbf{v}_i$, $t_i^{(n+1)} = t_i$, $\Gamma_i^{(n+1)} = \Gamma_i$, and $z_i^{(n+1)} = z_i$, $\forall i$;
11: Set $n = n+1$;
12: **until** $\Delta < \varepsilon$, where $\varepsilon > 0$ is the tolerance level.

Since the problem (16) is now a convex optimization problem for given $\{t_i^{(n)}\}$, $\{z_i^{(n)}\}$, $\{\Gamma_i^{(n)}\}$, and $\{\mathbf{v}_i^{(n)}\}$, we can readily solve it and update the values of the involved variables in the next iteration. Algorithm 2 outlines the proposed algorithm. By initially generating a feasible solution of (12) at $n=0$ and successively solving the approximated convex problem (16), the obtained solution approximates further accurately the optimum solution of the original MaxSum problem (11) in each iteration [10]–[12].

Because of (14) and (15), the optimal solutions obtained at the nth iteration for (16) are also feasible for the $(n+1)$th iteration. Thus, Algorithm 2 produces a nondecreasing sequence of objective values $\text{obj}^{(n)}$ in each iteration [11], i.e., $\text{obj}^{(n+1)} \ge \text{obj}^{(n)}$. Furthermore, the sequence of objective values $\text{obj}^{(n)}$ is bounded from above due to the power constraint (6c). Therefore, the convergence of Algorithm 2 is guaranteed.

C. Complexity Analysis

Algorithm 1 needs to solve N nonlinear equations and the power minimization problem (10) in each iteration. The complexity to solve the N nonlinear equations using the Newton-Raphson method is $O(N\log(\xi))$, where ξ is a constant depending on the required precision of the solution. If interior-point methods are applied to solve (10), the computational complexity is $O(N^{4.5}M^2 \log \epsilon_1^{-1})$, where ϵ_1 is the required accuracy of the duality gap termination [13]. Assume $\min_i d_i(r_i^{\text{sta}}) \le B$, where B is a constant. The required iteration steps of Algorithm 1 is $O(\log_2(B/\epsilon))$. Therefore, the total computational complexity of Algorithm 1 is $O((N^{4.5}M^2 \log \epsilon_1^{-1} + N\log(\xi))\log_2(B/\epsilon))$.

In Algorithm 2, a series of convex optimization problems (16) need to be solved. The problem (16) involves both exponential cone constraints (12c) and power cone constraints (12d). The exponential cone constraints (12c) can be approximated by second-order cones (SOCs) using a concave lower bound of the logarithmic function as in [12]. The power cone constraints (12d) can be converted to SOC constraints using the transformation as in [14, Section 3.3]. The computational complexity to solve the converted SOCP problem of (16) by using the interior-point methods is $O((N^{4.5}M^2 + N^{3.5}M^3)\log \epsilon_2^{-1})$, where ϵ_2 is its required accuracy of the duality gap termination [13]. Assume $L_{\max}$ iterations are performed in the worst case. The worst-case complexity of Algorithm 2 is then $O((N^{4.5}M^2 + N^{3.5}M^3)L_{\max}\log \epsilon_2^{-1})$.

In general, the theoretical computational complexity of Algorithm 2 is higher than that of Algorithm 1. This is because more auxiliary variables and constraints need to be considered in (16) than (10). Further results on the complexity comparison of the proposed algorithms will be given in the next section.

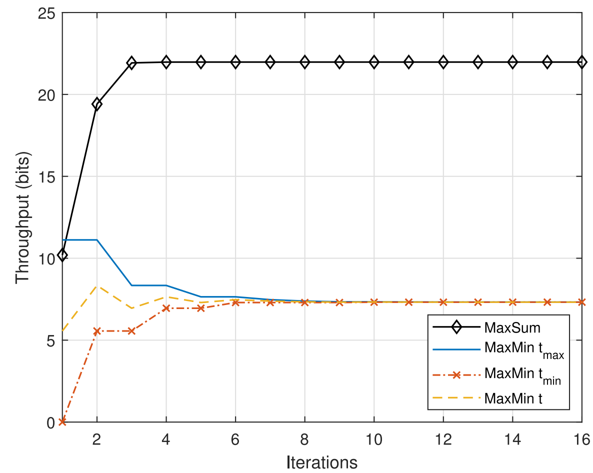

Fig. 2. Convergence comparison of data throughput for MaxSum and MaxMin schemes when $M=4$ and $N=3$.

IV. Simulation Results and Discussion

In the following simulations, power-related parameters are set as [2]: the fixed power consumption at each user $p_i = 0.3$ Watt, $\forall i$; the rate-dependent power consumption coefficient $b = 0.04$; the rate-dependent power consumption exponent $m = 1.2$; and the Peukert constant $a = 1.05$. The weights for the MaxSum problem are set to $w_i = 1$, $\forall i$, in the simulations.

The convergence results of the MaxMin scheme using Algorithm 1 and the MaxSum scheme using Algorithm 2 are shown in Fig. 2 for one channel realization. Here the number of transmitter antennas and users are four and three, respectively, i.e., $M=4$ and $N=3$. We observe that Algorithm 1 and 2 converge within approximately 10 and 5 iterations, respectively. In Algorithm 1, the value of $t_{\max}$ decreases while that of $t_{\min}$ increases monotonically as the number of iterations increases. In Algorithm 2, the objective value $\text{obj}^{(n)}$ increases monotonically until it converges.

The average total and per-iteration solving time comparison for the proposed algorithms is shown in Fig. 3, where the number of transmitter antennas is 12. The tolerance level ϵ in Algorithm 1 and ε in Algorithm 2 are both set to 10^{-4}. The simulations are performed on a computer with an Intel i7-6700 processor and the convex problems are solved with the MOSEK solver [15]. We observe that the per-iteration solving time for the MaxSum scheme is approximately 1.9 to 2.7 times that of the MaxMin scheme and the solving time for both schemes increases with the number of users. The average number of iterations for the MaxMin scheme is 17, which remains unchanged as N increases. The average number of iterations for the MaxSum scheme is between 5 and 41, which also increases with N. The simulation results of the solving time comparison conform to our theoretical complexity analysis.

For the average user throughput (denoted by "average") and the minimum user throughput (denoted by "min"), we compare the proposed

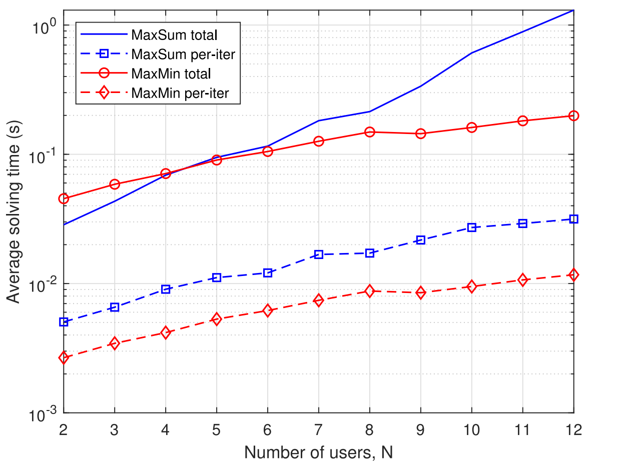

Fig. 3. Average total and average per-iteration solving time comparison when $M = 12$.

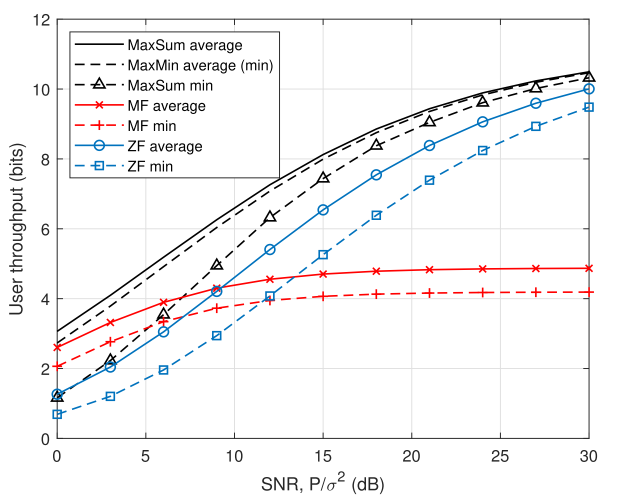

Fig. 4. Average and minimum user throughputs across SNR, when $M = 4$ and $N = 3$. Here $C_i U_i^a$ are normalized to 1.

schemes with two widely used multiuser schemes, i.e., the zero-forcing (ZF) and matched filtering (MF) transmission schemes [16]. We optimize the power allocation of each data stream for the ZF and MF schemes to maximize the minimum and sum of user data throughputs, when comparing them with the MaxMin and MaxSum schemes, respectively. The signal-to-noise ratio (SNR) is defined as P/σ^2 in the simulation. To obtain each point in the results, 500 independent channels are realized. Here, each complex channel element conforms to a standard normal distribution.

In Fig. 4, the average and minimum user throughputs across SNR are shown for the considered schemes when the transmitter has four antennas and there are three single-antenna users, namely, $M = 4$ and $N = 3$. The throughputs of all the considered schemes increase with SNR. We observe that the proposed schemes greatly outperform the ZF and MF schemes. This is due to the optimized joint design of beamforming and decoding data rates, which impacts the expected battery lifetime. For the average and minimum throughputs, the MaxSum and MaxMin schemes outperform the ZF scheme by about 5 dB and 8 dB, respectively. Furthermore, the ZF scheme increases much faster with the SNR compared to the MF scheme. This is because the ZF scheme

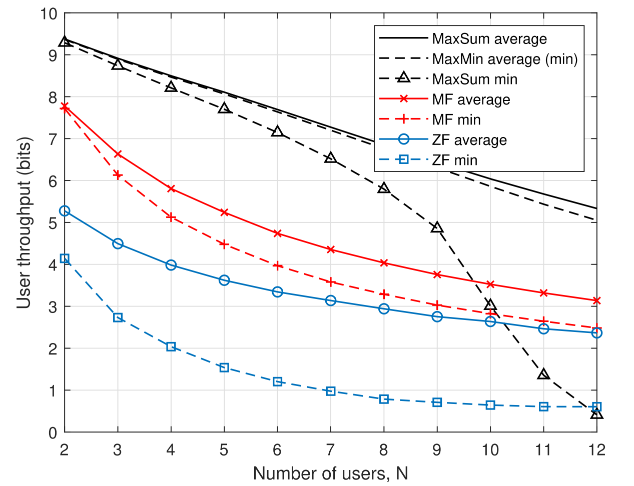

Fig. 5. Average and minimum user throughputs across N, when $M = 12$ and $P/\sigma^2 = 10$ dB. Here $C_i U_i^a$ are normalized to 1.

exploits the spatial degrees-of-freedom at high SNR and can achieve a higher throughput compared to the MF scheme, while the MF scheme suffers from the inter-user interference and its throughput saturates at high SNR. The MaxMin scheme has lower average throughput than the MaxSum scheme; however, it achieves higher minimum throughput compared to the MaxSum scheme. Since each user's throughput is upper bounded by $d_i(r_i^{\rm sta})$, the user throughput curve will eventually saturate as the SNR increases.

In Fig. 5, the average and minimum user throughputs are shown across the number of users when $M = 12$ and $P/\sigma^2 = 10$ dB. As expected, the user throughputs generally decrease with the number of users. Again, we observe that the MaxSum and MaxMin schemes still achieve the highest average and minimum throughputs, respectively. The MF scheme outperforms the ZF scheme in this result because the SNR in this scenario is medium. The impact of inter-user interference on the user side increases with the number of users N in the MF scheme. Therefore, the gap between the average throughputs of the MF and ZF schemes decreases with the increase of N. Since the MaxMin and MaxSum schemes take advantage of the joint design of beamforming and data rates, their average and minimum throughputs are more than 60% higher than those of the MF and ZF schemes, respectively.

V. Conclusion

In this paper, data throughput maximization problems during the expected battery lifetime are considered under multiuser environments. The battery lifetime is an important factor in the design of wireless networks. We formulated two problems that maximize either the minimum of user throughputs (MaxMin) or the weighted sum of user throughputs (MaxSum), and proposed efficient algorithms to solve both problems. Simulation results verified that the proposed algorithms can greatly improve the data throughput within the battery lifetime.

Appendix A
Proof of Remark 1

Without loss of generality, we assume $\min_i d_i(r_i^{\rm sta})$ is not jointly achievable by all users. Otherwise, the MaxMin throughput solution is simply $\min_i d_i(r_i^{\rm sta})$. In this case, we first denote the optimum data throughput of (7) as $t^{\rm opt}$. The corresponding data rates that achieves $t^{\rm opt}$ are denoted by $\{r_i^{\rm opt}\}$, where $d_i(r_i^{\rm opt}) = t^{\rm opt}, \forall i$. By definition, any data throughput t, where $t^{\text{opt}} < t \leq \min_i d_i(r_i^{\text{sta}})$, cannot be jointly achieved by all users. We then consider data throughput t, where $0 \leq t < t^{\text{opt}}$. Because the data throughput is monotonically increasing with the data rates in the range $[0, \min_i d_i(r_i^{\text{sta}})]$, the corresponding data rates $\{r_i\}$, such that $d_i(r_i) = t$, must satisfy $r_i \leq r_i^{\text{opt}}$, $\forall i$, according to Property 1. Since $\{r_i^{\text{opt}}\}$ is jointly achievable by all users, the data rate $\{r_i\}$ is also jointly achievable. Thus, any data throughput $t \in [0, t^{\text{opt}}]$ is achievable by all users. Therefore, by initializing $t_{\text{min}} = 0$ and $t_{\text{max}} = \min_i d_i(r_i^{\text{sta}})$, t_{max} and t_{min} in Algorithm 1 monotonically converge to t^{opt}. When ϵ is sufficiently small, the solution of t produced by Algorithm 1 is sufficiently close to t^{opt}.

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References

[1] F. Wang, C. Xu, L. Song, Q. Zhao, X. Wang, and Z. Han, "Energy-aware resource allocation for device-to-device underlay communication," in *Proc. IEEE Int. Conf. Commun.*, Budapest, Hungary, Jun. 2013, pp. 6076–6080.

[2] F. Wang, C. Xu, L. Song, and Z. Han, "Energy-efficient resource allocation for device-to-device underlay communication," *IEEE Trans. Wireless Commun.*, vol. 14, no. 4, pp. 2082–2092, Apr. 2015.

[3] H. M. Shin, S.-H. Park, J. Jung, S. Lee, and I. Lee, "Maximization of total throughput and device lifetime with non-linear battery properties," *IEEE Trans. Wireless Commun.*, vol. 16, no. 12, pp. 7774–7784, Dec. 2017.

[4] T. Wang and L. Vandendorpe, "On the optimum energy efficiency for flat-fading channels with rate-dependent circuit power," *IEEE Trans. Commun.*, vol. 61, no. 12, pp. 4910–4921, Dec. 2013.

[5] P. Rost and G. Fettweis, "On the transmission-computation-energy tradeoff in wireless and fixed networks," in *Proc. IEEE GLOBECOM Workshops*, Miami, FL, USA, Dec. 2010, pp. 1394–1399.

[6] R. Rao, S. Vrudhula, and D. N. Rakhmatov, "Battery modeling for energy-aware system design," *Computer*, vol. 36, no. 12, pp. 77–87, Dec. 2003.

[7] G. E. Forsythe, M. A. Malcolm, and C. B. Moler, *Computer Methods for Mathematical Computations*. Englewood Cliffs, NJ, USA: Prentice-Hall, 1977.

[8] A. Wiesel, Y. Eldar, and S. Shamai, "Linear precoding via conic optimization for fixed MIMO receivers," *IEEE Trans. Signal Process.*, vol. 54, no. 1, pp. 161–176, Jan. 2006.

[9] E. Visotsky and U. Madhow, "Optimum beamforming using transmit antenna arrays," in *Proc. IEEE Veh. Technol. Conf.*, Amsterdam, the Netherlands, Sep. 1999, pp. 851–856.

[10] T. Wang and L. Vandendorpe, "Successive convex approximation based methods for dynamic spectrum management," in *Proc. IEEE Int. Conf. Commun.*, Ottawa, ON, Canada, Jun. 2012, pp. 4061–4065.

[11] K.-G. Nguyen, Q.-D. Vu, M. Juntti, and L.-N. Tran, "Distributed solutions for energy efficiency fairness in multicell MISO downlink," *IEEE Trans. Wireless Commun.*, vol. 16, no. 9, pp. 6232–6247, Sep. 2017.

[12] K.-G. Nguyen, O. Tervo, Q.-D. Vu, L.-N. Tran, and M. Juntti, "Energy-efficient transmission strategies for CoMP downlink—Overview, extension, and numerical comparison," *EURASIP J. Wireless Commun. Netw.*, vol. 2018, no. 1, 2018, Art. no. 207.

[13] M. Lobo, L. Vandenberghe, S. Boyd, and H. Lebret, "Applications of second-order cone programming," *Linear Algebra Appl.*, vol. 284, no. 1, pp. 193–228, Nov. 1998.

[14] A. Ben-Tal and A. Nemirovski, *Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications*. Philadelphia, PA, USA: SIAM, 2001.

[15] MOSEK ApS, "MOSEK optimization software," 2019. [Online]. Available: https://www.mosek.com

[16] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication.*, Cambridge, U.K.: Cambridge Univ. Press, 2005.