

Multi-agent Formation Control with Target Tracking and Navigation

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Abstract— The formation control problem of a multi-agent system with target tracking and navigation is resolved by a gradient based extremum seeking control (ESC) associated with artificial potential functions methodology in this paper. It aims to maintain and achieve a stable formation for a swarm of multi-agent system, while guaranteeing tracking of a specified trajectory. By incorporation of artificial potential functions and extremum seeking control method, the potential function is minimized to achieve formation control and target tracking. The global trajectory is generated while the formation control and target tracking are carried out. A histogram-based local navigation integrated with map building is then applied to direct one of the agents to follow the planned trajectory to track the target while traversing with the rest of the agents. The simulation studies of multiple autonomous mobile robots are provided to demonstrate the effectiveness of the proposed hybrid control method.

I. INTRODUCTION

Formation control of multi-agent systems has increasingly drawn attention in the robotics, automation and control systems in recent years, given the fact that multi-agent formation control and target tracking have extensive applications such as surveillance, intelligent transportation, cooperative control of mobile vehicles, multi-vehicle robotic minesweeping, distributed moving sensor networks, etc. [1-2]. The objective of formation control with target tracking is to maintain and achieve a stable formation for a swarm of multiple agents, while tracking of target in a specified trajectory. A swarm of agents with even simple structure can fulfill more complex missions at less cost than a single complex robot because of their modularity and flexibility.

There have been a great number of studies conducted on multi-agent formation control using a variety of approaches such as output feedback [3], complex Laplacian [4-5], model independent coordination [6], feedback control [7] and extremum seeking control [9].

Zhou *et al.* [3] proposed an output feedback based method for the formation control of a high-order multi-agent system with directed communication topology, in which the time-varying formation control problem is converted to consensus problems and then stabilization problems. The sufficient conditions are achieved in light of Lyapunov stability analyses. However, the target tracking has not been taken into account. Lin *et al.* [4] developed a complex Laplacian based model to carry out the multi-agent formation control. The formation shapes specified by inter-agent relative

positions are specified in their model whereas with the proposed complex Laplacian, a distributed and linear control law forms the target formation shape. Jagtap *et al.* [5] integrated the complex Laplacian method with output regulation techniques based on consensus algorithm to synchronize motion of the multi-agent system. However, the tracking issue has not been resolved. Egerstedt and Hu [6] developed a model independent coordination approach for multi-agent formation control. Combined with a desired reference path for a virtual leader, the formation control problem is defined by a formation constraint. The tracking problem is resolved for a group of nonholonomic robots but there is no any navigation strategy that directs the mobile robots to traverse in the workspace. The relative distance and orientation of agents is exponentially stabilized by a feedback controller of a group of multiple robots in a leader-follower motion by only using local sensor-based information [7]. Arbitrarily large numbers of robots moving in general sorts of formations are controlled by their control law. However, in their simulation and experiments, the navigation component lacks in the motion control of six mobile robots.

Potential function based approach has been broadly used to resolve the target tracking and formation control issue [8]. By this approach, potential function is created to contain the scalar signal (the moving target) to be tracked, as well as the inter-connection between agents. The formation control and target tracking will be realized by minimizing the potential function. Extremum seeking control (ESC) method is to autonomously find an optimal system behavior for the closed-loop system, while simultaneously maintaining stability and boundedness of signals [9]. Gradient based ESC approach is a straightforward yet efficient method, in which, gradient information is utilized to seek extreme value. Due to the computational efficiency of gradient based ESC method, it suits to acquire the satisfactory control performance and analysis in the formation control with target tracking of multiple robots.

The formation control consists of the mixing of several missions. The first mission is the entire formation mission of moving along a predefined trajectory or moving along the center of the formation mass of a multi-agent system. The second mission, in order for the shape to be preserved, is to sustain the relative positions of the multiple agents during formation motion. The third is to avoid obstacles while the multiple agents are directed to traverse along the trajectory that is fulfilled by the local navigator. A fourth might be to perform the target tracking with the local navigation.

In this paper, the formation control problem of a multi-agent system with target tracking and navigation is resolved by a gradient based ESC and artificial potential functions methodology. The potential functions are minimized to achieve formation control and target tracking. The multiple agents traverse along a predefined trajectory with target tracking while maintaining the relative positions of the agents

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so that the shape is preserved. A histogram-based local navigation with map building is performed to direct the collision-free motion.

II. MULTI-AGENT FORMATION CONTROL PROBLEM FORMULATION

In this section, the multi-agent formation control issue will be addressed.

A. The Model of a Multi-agent System

A multi-agent system with N agents is involved in this section. The tracking problem includes finding an effective control approach for a group of agents, in order to make them achieve and maintain a given geometry. At the same time, the agents need to follow a moving target or a mobile signal.

A multi-agent system consists of N robots in n dimension Euclidean space. Let $x^i \in \mathbb{R}^n$ represents the position of the i th single agent; let $x^T = [x_1^T, \dots, x_N^T] \in \mathbb{R}^{n \times N}$. Assume the i th agent's dynamic is point-mass kinematic model

$$\dot{x}^i = u^i, \quad (1)$$

where $u^i \in \mathbb{R}^n$ represents the control input of the i th agent.

B. Extremum Seeking Control

The block diagram of extremum seeking control and the multi-agent system is illustrated in Fig 1. Firstly the Extremum seeking control algorithm needs a performance function which can be minimized to achieve the control goal. Secondly we need to design a Extremum Seeking Controller to compute the control input u to control the system and drive the performance output to a new state x . After a finite iteration the control method can get the extremum of the performance function [9].

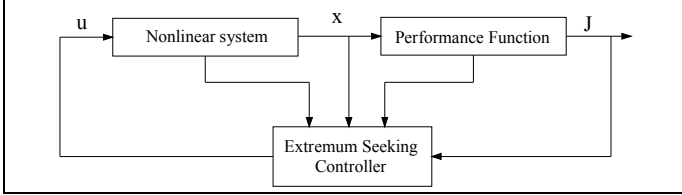


Figure 1 General block diagram for extremum seeking control

C. Potential Function based Approach

There are a number of different approaches to achieve the formation control including behavior based approach, potential field approach, leader-follower approach, virtual structure approach, cyclic approach, model predictive control approach and distributed approach etc. [10].

The potential function comprised of two parts is considered in this paper. First, the distance constraint enforces a restrict on the each agents, based on its adjacent agent's positions, so as to maintain a given geometrical formation. This component includes functions of the relative distance between every two adjacent agents. The second, the distance

between the scalar signal and the agents enforces a tracking component into the potential function. This part is to direct the multi-agent's behavior to follow the moving target. This tracking part is an artificial potential function according to the knowledge of target position. All in all we define the form of the potential function in light of the expected geometric formation shape.

By using artificial potential functions to encode the agent-target and agent-agent interaction, the extremum seeking techniques are employed for controller design of each agent, which is decentralized and in some instances the knowledge of target position and agent positions is not required [11]. However, by appropriate selection of the potential function one can always guarantee that eventually the moving target will be tracked and surrounded with the expected geometric formation by the tracking agents.

D. Build a Potential Function for Multi-agent Formation Control

The potential function between agent and the moving target is defined as

$$J_{at}(x, x_t) = \sum_{i=1}^N J_{it}(\|x^i - x_t\|) \quad (2)$$

where J_{it} is the potential function between the i th agent and the moving target. We assume J_{it} obtain the minimum at $\|x^i - x_t\| = \delta_{it}$. Where δ_{it} means the predefined distance between the agent and another and $\delta_{it} \neq 0$ to avoid the collision [10].

The potential function between one agent and another agent is

$$J_{aa}(x) = \sum_{i=1}^{N-1} \sum_{j=i+1}^N J_{ij}(\|x^i - x^j\|) \quad (3)$$

where, $J_{ij}(\|x^i - x^j\|)$ is the first part of the potential function represents the distance constraint between the i th agent and the j th agent. Let

$$\lim_{t \rightarrow \infty} \left\| \|x^i - x^j\| - d_{ij} \right\| < \varepsilon$$

where d_{ij} is the distance between the agent and the adjacent agent.

Addition of the above functions, leads to the following equation.

$$\begin{aligned} y &= J(x, x_t) \\ &= J(\|x^i - x_t\|, \|x^i - x^j\|), \quad 1 \leq i, j \leq N \\ &= K_{at} J_{at}(x, x_t) + K_{aa} J_{aa}(x) \\ &= K_{at} \sum_{i=1}^N J_{it}(\|x^i - x_t\|) + K_{aa} \sum_{i=1}^{N-1} \sum_{j=i+1}^N J_{ij}(\|x^i - x^j\|). \end{aligned} \quad (4)$$

where, y represents the whole potential function (performance function), K_{at} is the weight of the potential function between the agent and the moving target, K_{aa} is the weight of potential function which represents the distance

from one agent to another. Building a potential function makes the objective function reach the minimum at $\|x^i - x^j\| = \delta_{ij}$ and $\|x^i - x^j\| = d_{ij}$. As long as extremum seeking controller of each agent can minimize the objective function, we can achieve track the moving target, formation control, and obstacle avoidance while a local navigation is carried out.

III. SLIDING MODE CONTROL FOR GRADIENT BASED EXTREMUM SEEKING METHOD

Extremum seeking control algorithm applied in a multi-agent system is illustrated in Fig 2. There are N agents in the multi-agent system to be controlled.

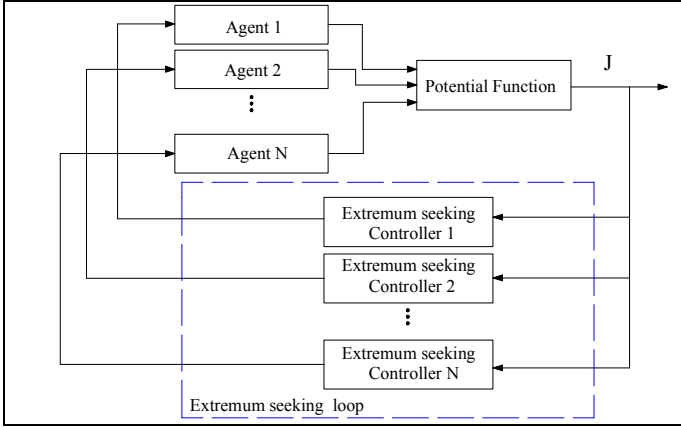


Figure 2 multi-agent tracking for N agents/vehicles

Gradient based extremum seeking control is used to control each agent by assuming the potential function J holds for Assumption A1-A5 [10] as follows.

Assumption A1: For $i=1, \dots, N$, there exists functions $h^i: \mathbb{R}^+ \rightarrow \mathbb{R}$ such that

$$\nabla_{x^i} J_{ii}(\|x\|) = x h^i(\|x\|). \quad (5)$$

Assumption A2: There exist unique distance δ_{ii} at which we have

$$h^i(\|\delta_{ii}\|) = 0. \quad (6)$$

Assumption A3: The potentials $J_{ij}(\|x^i - x^j\|)$ are symmetric and satisfy

$$\nabla_{x^i} J_{ij}(\|x^i - x^j\|) = -\nabla_{x^j} J_{ij}(\|x^i - x^j\|). \quad (7)$$

Assumption A4: For $1 \leq i, j \leq N$ there exist functions $g_{ar}^{ij}: \mathbb{R}^+ \rightarrow \mathbb{R}$ such that

$$\nabla_{x^i} J_{ij}(\|x\|) = x g_{ar}^{ij}(\|x\|). \quad (8)$$

Assumption A5: There exist unique distances δ_{ij} at which we have

$$g_{ar}^{ij}(\|x\|) \begin{cases} > 0, & \|x\| > \delta_{ij}, \\ = 0, & \|x\| = \delta_{ij}, \\ < 0, & \|x\| < \delta_{ij}. \end{cases} \quad (9)$$

The potential function which satisfying the above Assumptions A3 to A5 are odd functions. The term $g_{ar}^{ij}(\|x\|)$ in Assumption A5 represents the attraction-repulsion relationship between the agents and another.

Take the derivative of equation (4) we can acquire the following results

$$\nabla_{x^i} J(x, x_t) = K_{at}(x^i - x_t) h^i(\|x^i - x_t\|) + K_{aa} \sum_{j=1, j \neq i}^N (x^i - x^j) g_{ar}^{ij}(\|x^i - x^j\|). \quad (10)$$

$$\nabla_{x^i} J(x, x_t) = -K_{at} \sum_{i=1}^N (x^i - x_t) h^i(\|x^i - x_t\|). \quad (11)$$

From equalities in (6) and (7), we rewritten as

$$\nabla_{x^i} J(x, x_t) = -\sum_{i=1}^N \nabla_{x^i} J(x, x_t) + K_{aa} \sum_{i=1}^N \sum_{j=1, j \neq i}^N (x^i - x^j) g_{ar}^{ij}(\|x^i - x^j\|). \quad (12)$$

Besides, since we have

$$\sum_{i=1}^N \sum_{j=1, j \neq i}^N (x^i - x^j) g_{ar}^{ij}(\|x^i - x^j\|) = 0 \quad (13)$$

from the above Assumption 3, we get the following results

$$\nabla_{x^i} J(x, x_t) = -\sum_{i=1}^N \nabla_{x^i} J(x, x_t) \quad (14)$$

Choose a Lyapunov function as $V = J(x, x_t)$. Then we can get the following results

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N [\nabla_{x^i} J(x, x_t)]^T \dot{x}^i + [\nabla_{x^i} J(x, x_t)]^T \dot{x}_t \\ &= \sum_{i=1}^N [\nabla_{x^i} J(x, x_t)]^T (u^i - \dot{x}_t), \end{aligned}$$

So for the i th agent, we employ the control input as

$$u^i = \dot{x}_t - k^i \nabla_{x^i} J(x, x_t) \quad k^i > 0. \quad (15)$$

Then we obtain

$$\dot{V} = -\sum_{i=1}^N k^i \|\nabla_{x^i} J(x, x_t)\|^2 \leq 0.$$

If we assume that $\|\dot{x}_t\| \leq \gamma_t$ for some known $\gamma_t > 0$ [10]. Let $k^i > 0, \beta^i > \gamma_t$ and the controller can be formulated as

$$u^i = -(k^i + \beta^i) \text{sgn}(\nabla_{x^i} J(x, x_t)). \quad (16)$$

where $\text{sgn}(\bullet)$ is the signum function. Then, we obtain

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N (-(k^i + \beta^i) \|\nabla_{x^i} J(x, x_t)\| - [\nabla_{x^i} J(x, x_t)]^T \dot{x}_t) \\ &\leq -\sum_{i=1}^N k^i \|\nabla_{x^i} J(x, x_t)\|^2 \leq 0. \end{aligned}$$

From Lyapunov stability theory and the sliding mode control theory, with this control input the control goal will be achieved.

IV. LOCAL NAVIGATION/ROBOT NAVIGATION

In this paper, a Vector Field Histogram (VFH) based algorithm is utilized to serve as the local navigator with map building to assist the robots to perform collision-free local navigation, in which one of the agents/robots is directed to traverse along the predefined trajectory. The VFH method employs a polar histogram, a kind of intermediate data-representation to perform the local navigation. In the formation control, in order for the multiple robots to move along the predefined trajectory, the robot local navigation is required using this kind of VFH navigation algorithm. The VFH algorithm provides robots with a sufficiently detailed spatial representation of the environments ([12]). To avoid obstacles, the VFH serves as a local reactive navigation algorithm based directly on the sensor data (LIDAR, SONAR, GPS, etc.). In this paper, this polar histogram is divided into 54 sectors as every sector is 5° and the LIDAR has a range of 270° ([12]). The selected sector to go through is then utilized to guide the vehicle based on a weighted formula that combines deviation from desired direction and associated obstacle densities. The VFH simulation development environment was based on Player/Stag ([13]). Stage provides a powerful simulation environment that can be used to develop and test algorithms. The simulation result using VFH navigation algorithm is shown in Fig 3.

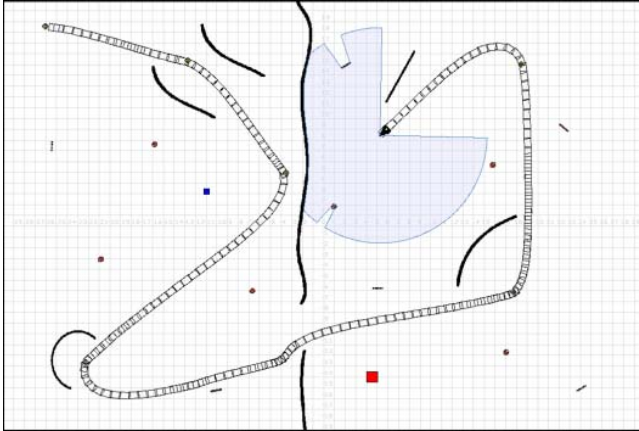


Figure 3 Simulation result of robot navigation by the VFH algorithm

V. SIMULATION STUDIES

In this section, we successfully implement the above developed control scheme to the formation control with target tracking and navigation of a team of three mobile robots. Three robots will form a triangular shape and meanwhile the control scheme can guarantee to track an arbitrarily scalar signal. Formation control with target tracking and local navigation is presented as follows by simulations of three mobile robots. For simulation, the agents and the target are involved in \mathbb{R}^2 with the target dynamics

$$\begin{cases} \dot{x}_{t1}(t) = 0.25, \\ \dot{x}_{t2}(t) = \cos(0.25t) \end{cases} \quad (17)$$

This target dynamics are not necessarily known. Through the algorithm presented in this paper the mission of target tracking, formation control and navigation will be fulfilled. The five-pointed star represents the signal of the moving target, whereas the circles in triangle represent the three robots as illustrated in Fig 4, in which the three robots keep equilateral triangle geometry and at the same time tracking a moving target. We assume in the first 25 seconds the distance between one robot and another is $\sqrt{3}$ and the distance between the robot and the moving target is 1. The last 25 seconds the distance between the one robot and another is 1 and the distance between the robot and the moving target is $\sqrt{3}/6$, respectively.

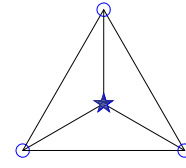


Figure 4 The desired formation shape

In order to achieve the above objective, the potential function is defined as follows

$$J_{at}(x, x_t) = \sum_{i=1}^3 \frac{1}{2} (\|x^i - x_t\|^2 - \delta_{it}^2)^2 \quad (18)$$

$$J_{aa}(x) = \sum_{i=1}^2 \sum_{j=2}^3 \frac{1}{2} (\|x^i - x^j\|^2 - \delta_{ij}^2)^2 \quad (19)$$

$$y = J(x, x_t) = K_{at} \sum_{i=1}^3 \frac{1}{2} (\|x^i - x_t\|^2 - \delta_{it}^2)^2 + K_{aa} \sum_{i=1}^2 \sum_{j=2}^3 \frac{1}{2} (\|x^i - x^j\|^2 - \delta_{ij}^2)^2. \quad (20)$$

where, $K_{at} = 0.5$, $K_{aa} = 1$, and let $\delta_{it} = 1$, $\delta_{ij} = \sqrt{3}$ in the first 25 seconds and $\delta_{it} = \sqrt{3}/6$, $\delta_{ij} = 1$ in the last 25 seconds respectively.

Simulation result is shown in Fig 4, the three robots keep an equilateral triangle geometry, and tracking a moving target which is changing as a cosine function. It is successfully achieve the goal.

Simulation result for the formation of the robots while tracking the moving target is shown in Fig 5. Three mobile robots track a moving target, in the first 25s the distance between each robot is $d_{12} = d_{23} = d_{31} = \sqrt{3}$, and the distance between the robot to the target d_{at} is 1. In the last 25s the distance becomes to $d_{12} = d_{23} = d_{31} = 1$ and d_{at} is $\sqrt{3}/6$. The triangle drawn by dashed line represents the robots' current geometrical position. The Euclidean inter-robot distance over the time in the simulation is illustrated in Fig 5. It is clear that over the time the distance between robots d_{ij} always agrees the desired value. The error of absolute inter-robot distance depicted in Fig 7. The absolute inter-robot distance errors converge to zero implies that the proposed model fulfills the formation control while maintaining a triangle shape and tracking the moving target. The performance function along

the simulation time is illustrated in Fig 8. The value of the performance function becomes zero at the first few seconds and only a slight pulse at the 25 seconds that assures the success of formation control.

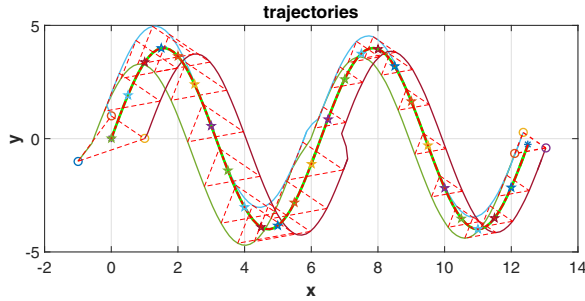


Figure 5 Three robots tracking a moving target, in the first 25s the distance between each agent is $d_{12}=d_{23}=d_{31}=\sqrt{3}$, and the distance between the agent to the target d_{at} is 1 and the last 25s the distance become to $d_{12}=d_{23}=d_{31}=1$ and d_{at} become $\sqrt{3}/6$. The five-pointed star represent moving target trajectory, three robots keep an equilateral triangle geometry and tracking the moving target with a predefined distance

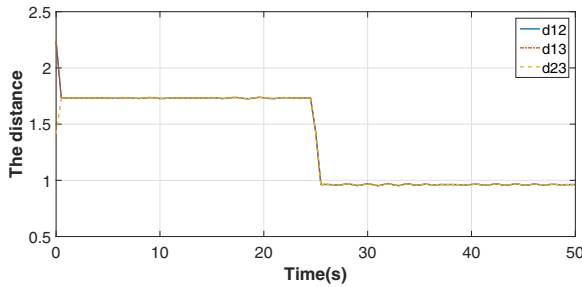


Figure 6 The distance between any two robots, $i = 1, 2, 3$. The desired distance is $d_{12}=d_{23}=d_{31}=\sqrt{3}$ in the first 25s and $d_{12}=d_{23}=d_{31}=1$ in the last 25s

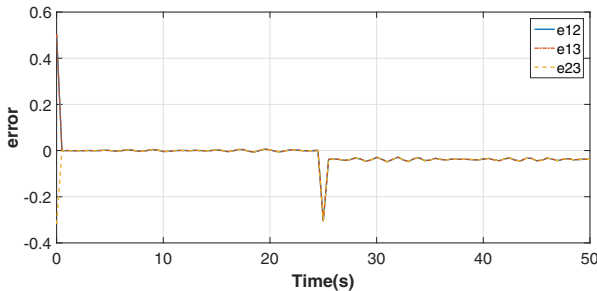


Figure 7 The error of the distance between each robots

$$e_{ij} = \lim_{t \rightarrow \infty} \|x^i - x^j\| - d_{ij}, i = 1, 2, 3.$$

As described previously, a necessary local navigation is implemented collision-free motion. Three mobile robots traverse in the workspace during the formation control by following the trajectory in Fig. 5, which is achieved by the VFH local navigator to achieve a collision-free motion. It is assumed, in the simulation studies that one of robots serves as the leader, who is directed by the VFH algorithm (the VFH local navigation algorithm is readily extended to three robots).

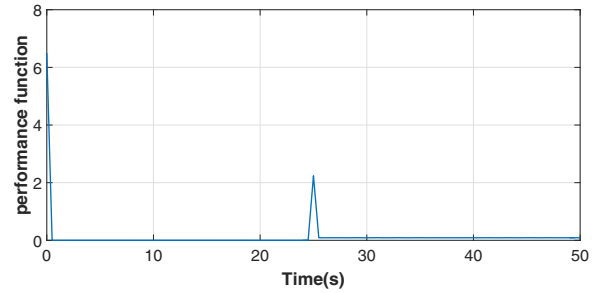


Figure 8 Evolution of the performance function $y=J(x,x_i)$.

As the robot moves to track the moving target, the coordinates of trajectory can be stored as markers that are sent to the VFH algorithm. The markers are depicted by “X” sign (the pink trajectory is produced by the one of the robots, the leader, for instance). Three robots are represented by solid circles in Fig. 9. In the navigation algorithm, one robot regards the rest of robots and the moving target as obstacles [15], in which the obstacle avoidance component in the VFH algorithm functions shown in Fig. 10.

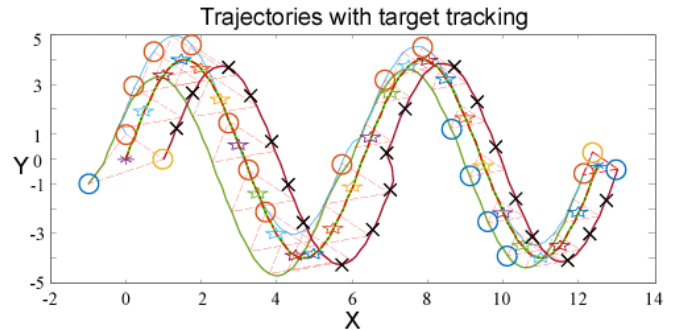


Figure 9 Markers in the trajectory

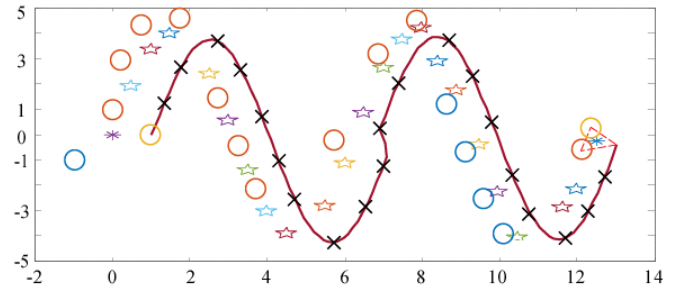


Figure 10 Incorporation of the local navigation

The mobile robot traverses from the initial position during the formation control to track the moving target. As the robot tracks the moving target, the local VFH navigator is necessary to perform the navigation to direct the robot along the trajectory to move. The VFH algorithm drives the robot from one marker to another by the point-to-point navigation algorithm. The early stage of navigation is illustrated in Fig. 11 whereas the stage that the robot nearly completes the collision-free motion is shown in Fig. 12. A local map composed of square grids is created through the local navigator while the robot traverses with LIDAR information. From the measured sensory information, a map of the robot’s

immediate limited surroundings is dynamically built for navigation. The global map is formed by incorporation of every local map built locally by LIDAR-based sensor. In the last stage the built map locally by LIDAR-based mapping model is imaged in Fig. 12.

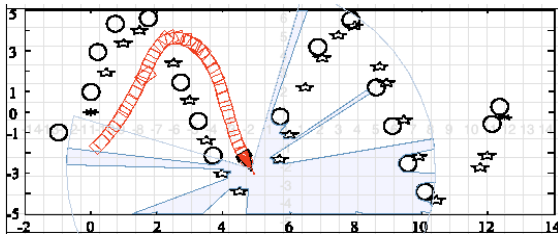


Figure 11 The early stage of the navigation of the robot

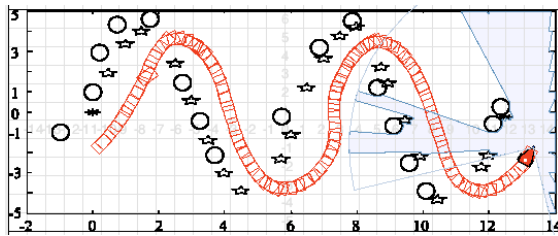


Figure 12 The fulfilled navigation of the robot

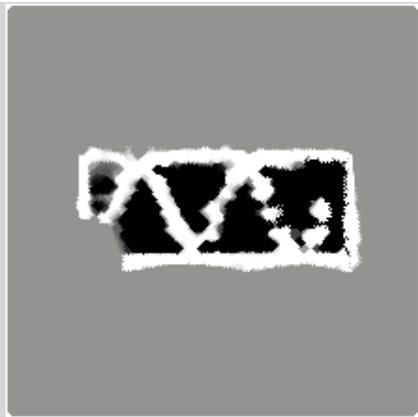


Figure 13 The constructed map of the robot

VI. CONCLUSION

In this paper a hybrid method to the formation control problem of a multi-agent system while tracking a moving target with navigation is proposed. The local navigation algorithm has been taken into consideration of the gradient based extremum seeking control algorithm. The formation control with target control is achieved by minimizing the potential function by the extremum seeking control algorithm. The proposed hybrid control strategy is successfully capable of minimizing the performance function in a short period of time for the multiple agents to maintain a given geometrical formation while simultaneously tracking the target signal. One of the mobile robots as the leader can be guided by the VFH local navigator to follow the

collision-free trajectory. The effectiveness and robustness of the proposed hybrid model has been demonstrated by our simulation studies.

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